Walking through the Cloud: Routing of Virtualized Network Functions

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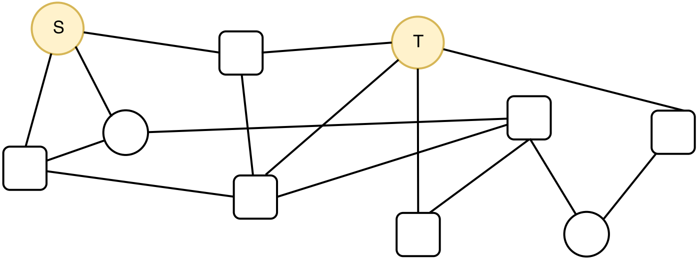
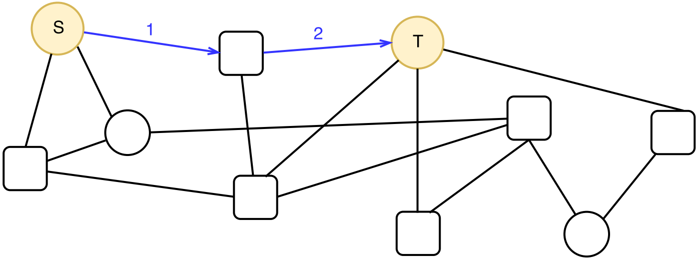
A Routing Problem



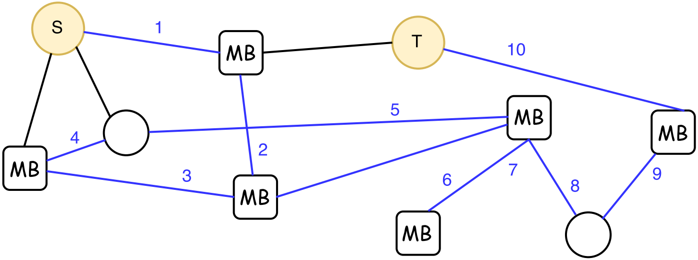
A Routing Problem
the usual routing



A Routing Problem

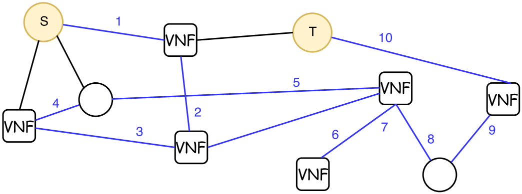
new: middleboxes

- Middlebox: Firewall, NAT, proxies, DPI etc.



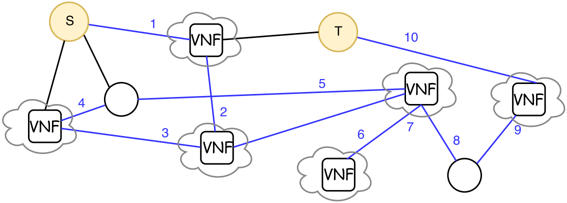
A Routing Problem
moving to clouds

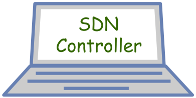
- VNFs brings flexibility, are cheaper



A Routing Problem
moving to clouds

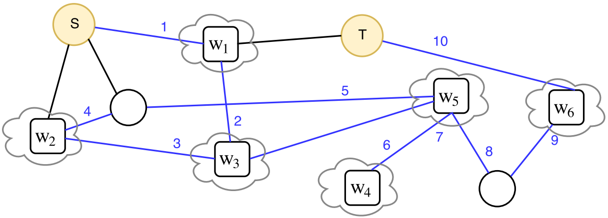
- VNFs brings flexibility, are cheaper
- A lot of them, in clouds

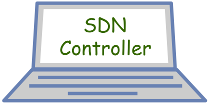




A Routing Problem
the waypoints

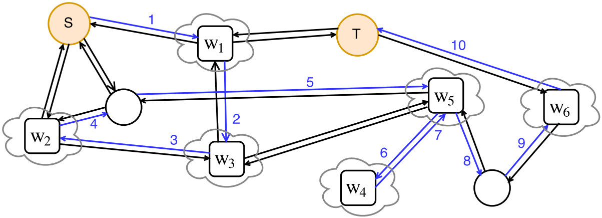
- The task: find the shortest S–T walk through *waypoints*
- Capacities must be respected

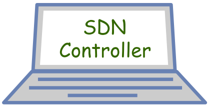




A Routing Problem
the network

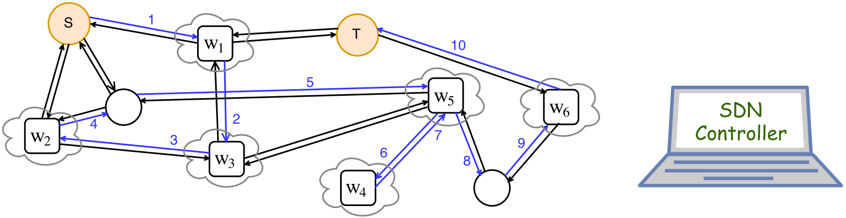
- Real Networks Are Bidirected





A Routing Problem
in two flavors

- Real Networks Are Bidirected
- Two Flavors: Ordered vs Unordered



Outline

Model

- Bidirected graph $G(V, E)$: $\forall (x, y) \in E \implies (y, x) \in E$
- n nodes, k of which are waypoints
- Arbitrary capacities, unit demand for (S, T)

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- The shortest *feasible* S-T walk visiting *all* waypoints
- A feasible walk respects link capacities

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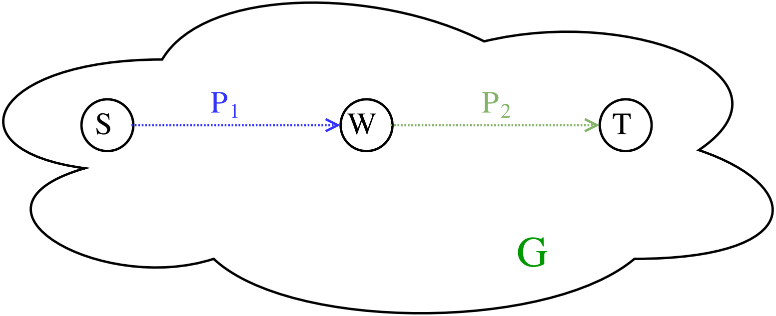
- Ordered and Unordered

Outline

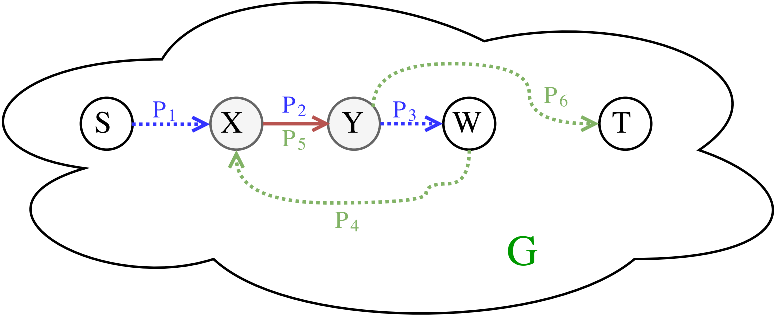
One waypoint: greedy is optimal

Two shortest paths?



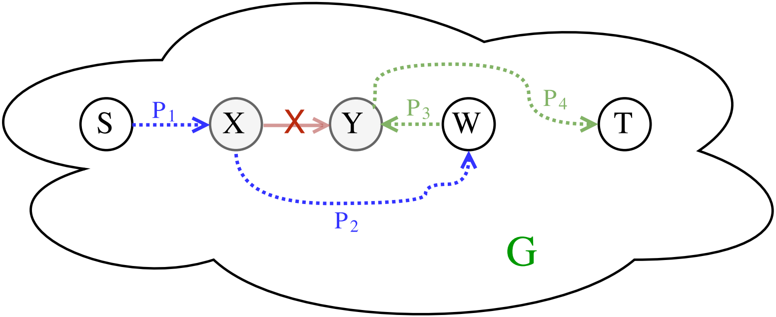
One waypoint: greedy is optimal

Assume both shortest paths chose (X, Y)



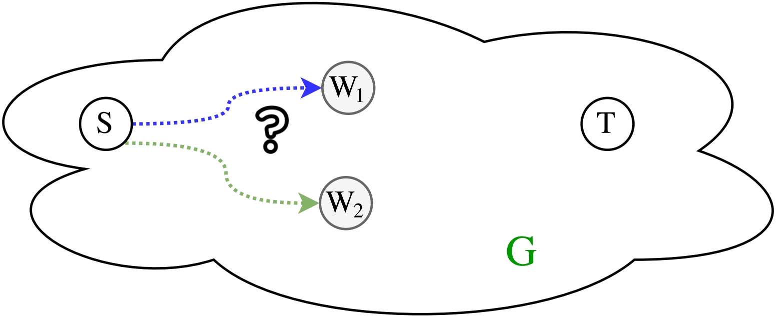
One waypoint: greedy is optimal

$P_1 P_2 P_3 P_4$ is shorter $\implies$ Contradiction!



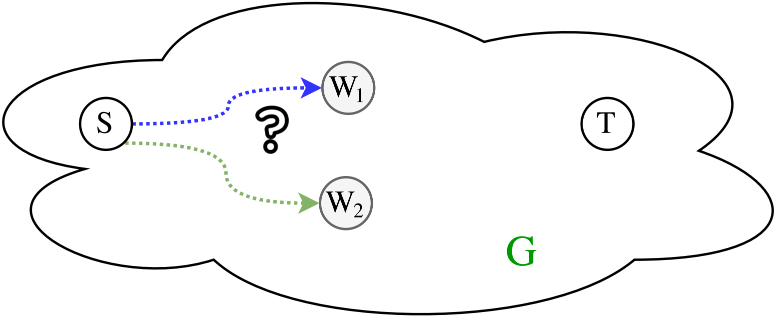
More waypoints

✓ The optimal order + shortest paths $\Longrightarrow$ it works!



More waypoints

- ✓ The optimal order + shortest paths $\implies$ it works!
- ✓ Try all permutations: SW_1W_2T or SW_2W_1T?



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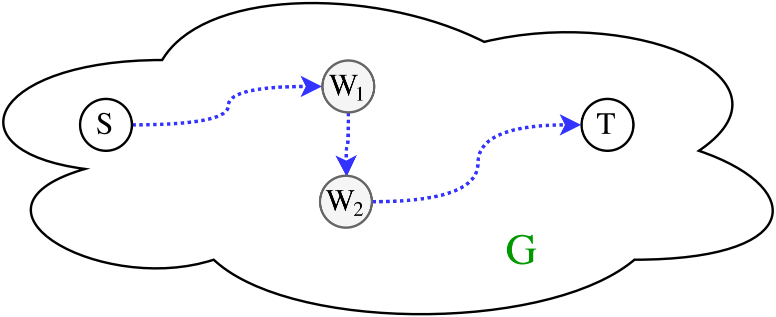

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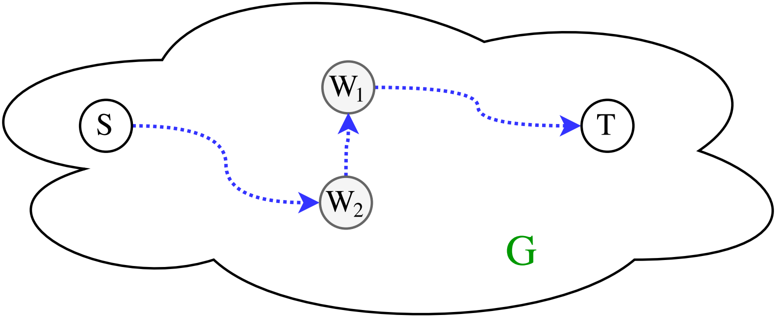
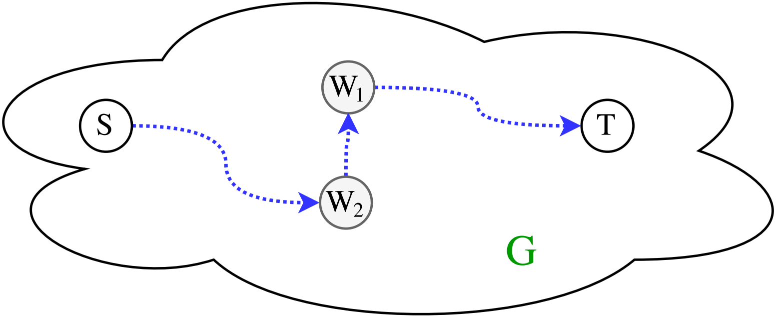


More waypoints

- ✓ The optimal order + shortest paths $\implies$ it works!
- ✓ Try all permutations: SW_1W_2T or SW_2W_1T?
- ✓ Polynomial time for $k = \mathcal{O}\left(\frac{\log n}{\log \log n}\right)$



Outline

Hardness

- Feasibility via spanning tree $\implies$ always feasible

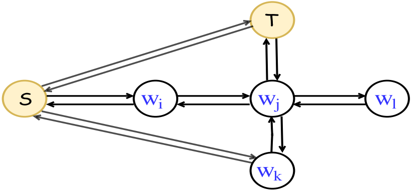


Figure: Bidirected

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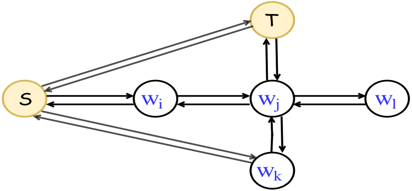

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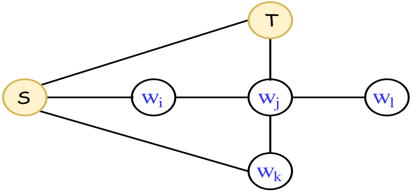

Figure: Undirected

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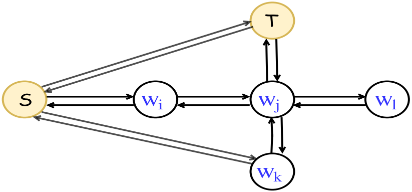

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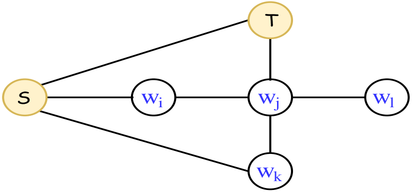

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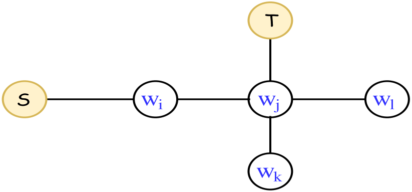

Figure: Spanning Tree

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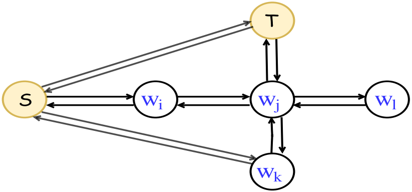

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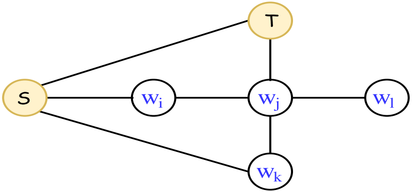

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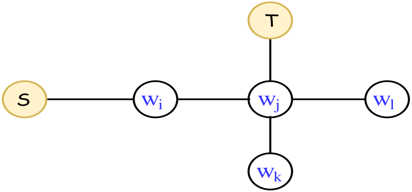

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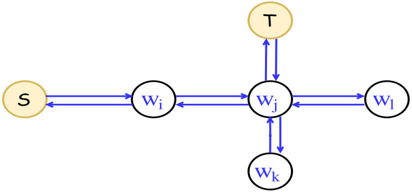
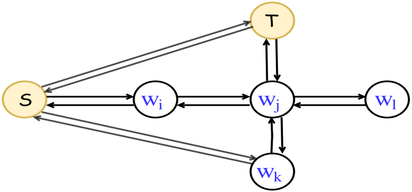


Figure: Bidirected again

Hardness

- Feasibility via spanning tree $\Longrightarrow$ always feasible



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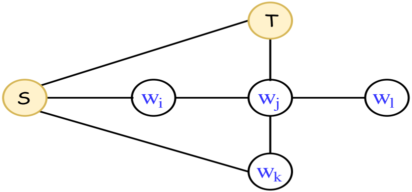

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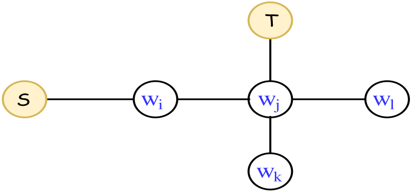

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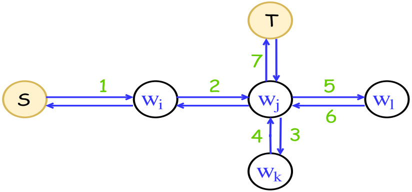

Figure: S-T tour

Hardness

- Feasibility via spanning tree $\implies$ always feasible
- Approximation via metric TSP $\implies$ L: $\approx$1.008[1], U: $\approx$1.53[2]

[1] Karpinski et al. J. Comput. Syst. Sci., 2015
[2] Andr as Seb o and Anke van Zuylen, FOCS 2016

Hardness

- Feasibility via spanning tree $\implies$ always feasible
- Approximation via metric TSP $\implies$ L: ≈ 1.008[1], U: ≈ 1.53[2]
- FPT via subset TSP $\implies 2^k \cdot n^{\mathcal{O}(1)}$ (Klein and Marx, 2014)

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Outline

Ordered waypoint routing
The problem

- A permutation is given, e.g. $w_1 w_2 \ldots w_k$
- Find the shortest route visiting every w_i, satisfying the permutation

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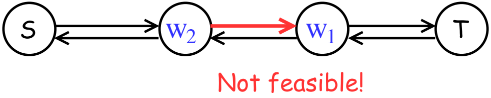

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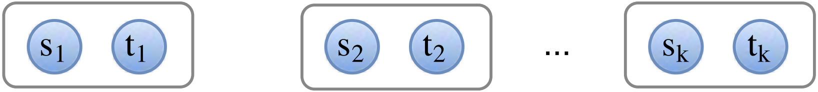
- Related to **E**dge **D**isjoint **P**ath **P**roblem
- NP-Hardness and feasibility via EDPP

Edge Disjoint Path Problem
EDPP

Edge Disjoint Path Problem $\in$ NP-Complete

Find a set of pairwise edge-disjoint paths connecting every pair $(s_i, t_i), i = 1 \ldots k$

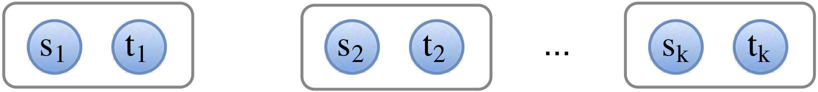


Reduction
from EDPP

Building the OWRP instance

The waypoints:

$s_1, t_1 ... s_i, t_i, s_{i+1}, t_{i+1} ... s_k, t_k$



Reduction
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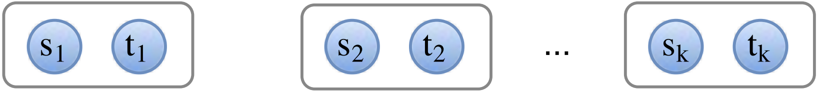
Building the OWRP instance

The waypoints:
$S = s_1, t_1 \ldots s_i, t_i, s_{i+1}, t_{i+1} \ldots s_k, t_k = T$



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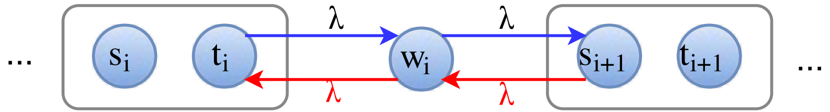

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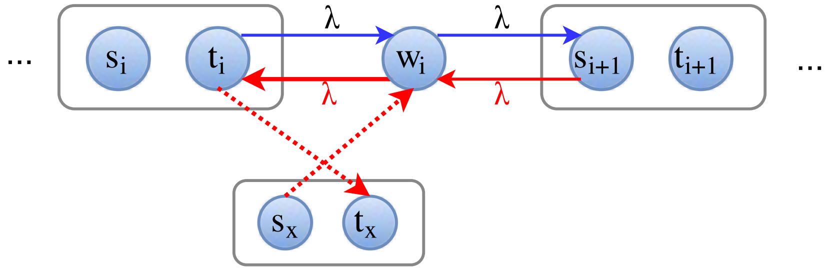

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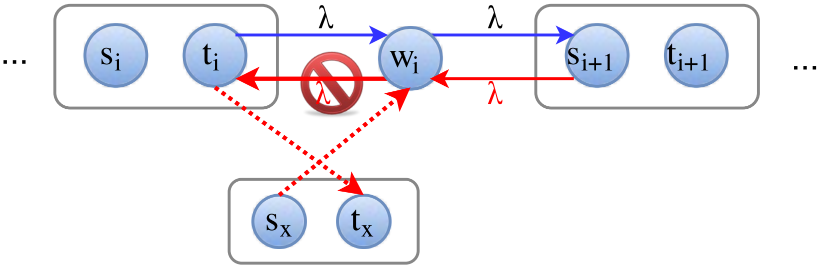

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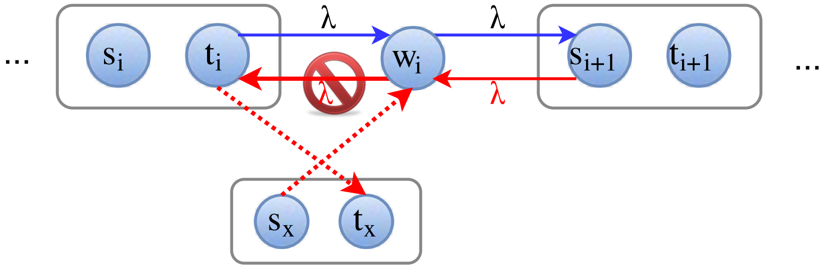

- Set λ large enough

Reduction
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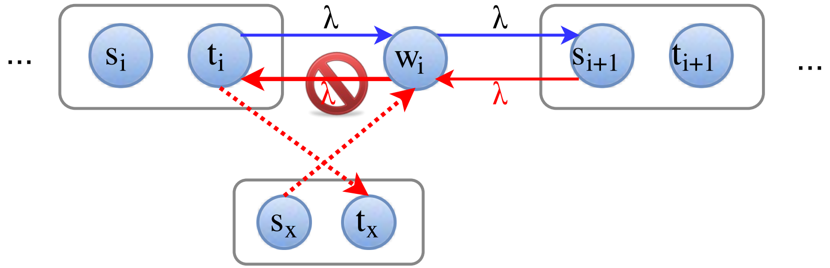

- Set λ large enough
- OWRP chooses a backward edge $\iff$ EDPP is not feasible

Ordered waypoint routing

Results

- General graphs: $k \in \mathcal{O}(1) \implies$ feasibility $\in$ P, via EDPP (A. Jarry et al., 2009)

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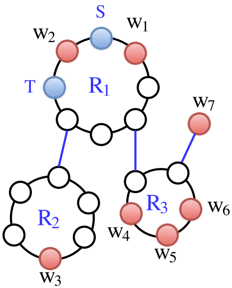
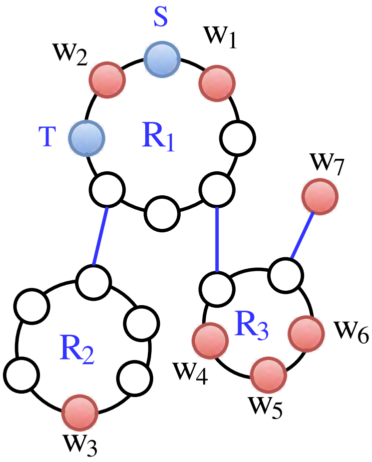
- *Cactus graph*: tree of rings



Ordered waypoint routing
Cactus: a tree of rings

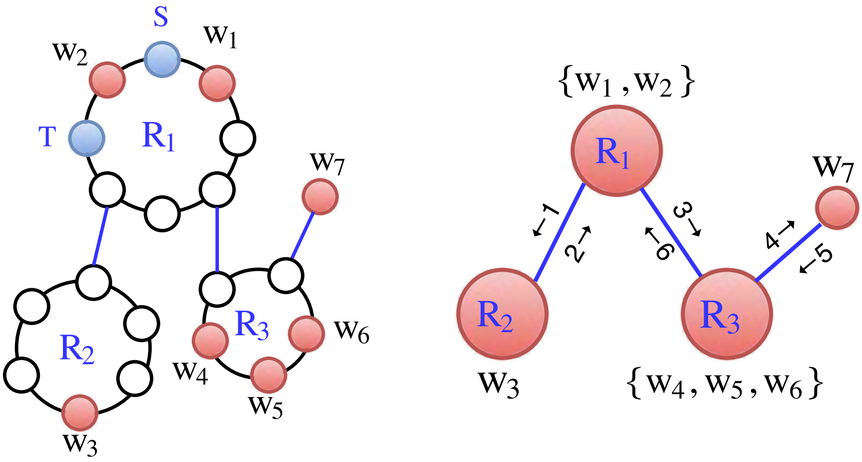


Step 1: solve the tree contraction given

$(S = R_1), R_2, R_3, W_7, (R_1 = T)$

Ordered waypoint routing

Cactus: a tree of rings



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Ordered waypoint routing

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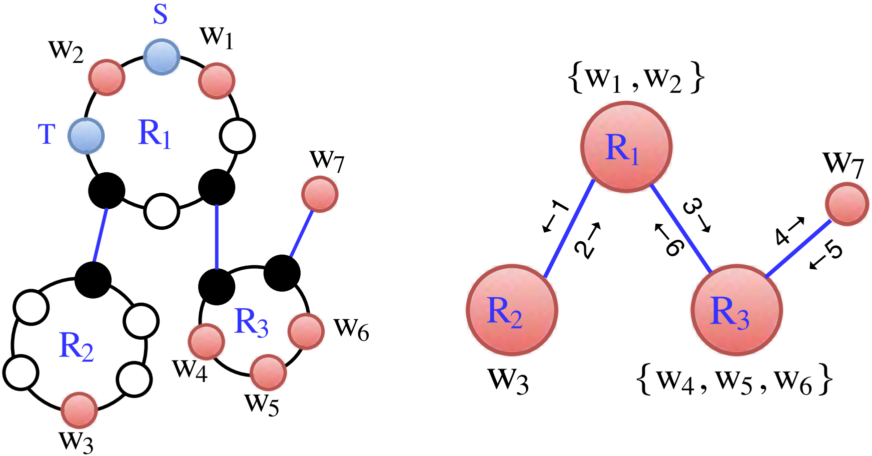

- Step 2: mark the port nodes (shown in back) as new waypoints

Ordered waypoint routing

Cactus: a tree of rings

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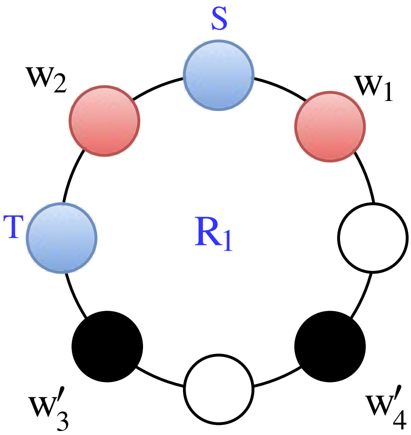
- Step 3: solve OWRP on each ring separately

Ordered waypoint routing
Cactus: a tree of rings



Ordered waypoint routing

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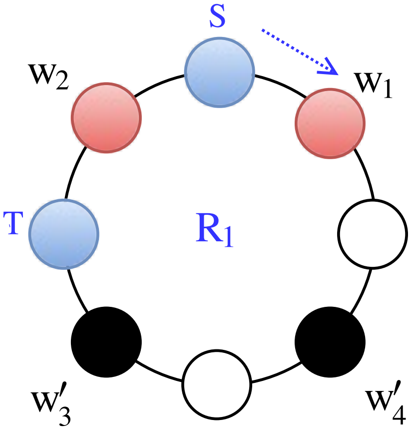
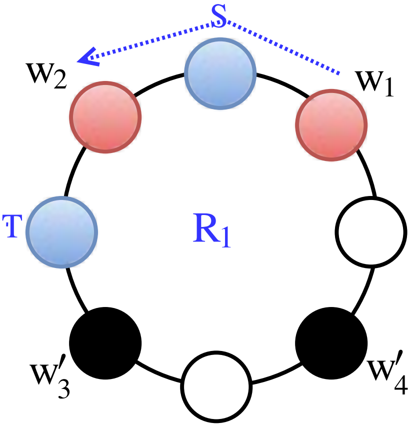


Ordered waypoint routing

Cactus: a tree of rings



Ordered waypoint routing

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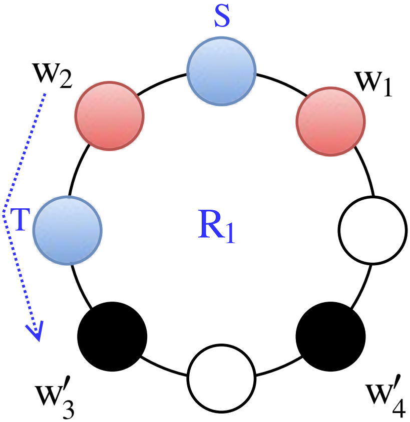

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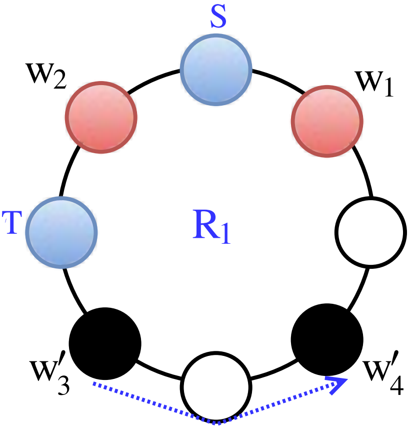
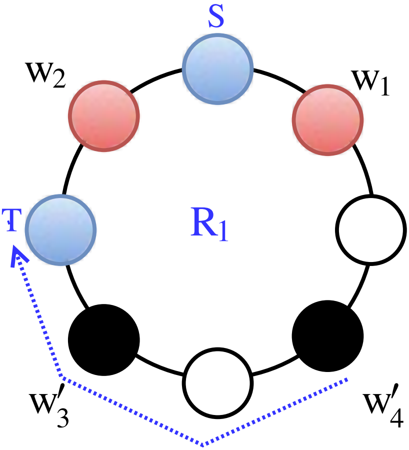


Ordered waypoint routing

Cactus: a tree of rings



Summary

Table: Ordered WRP

	General	$k \in \mathcal{O}(1)$	Tree	$c_e \in \mathcal{O}(1)$
Feasibility	open	P	P	Ring $\in$ P
Optimality	NP-Hard	open		

Table: Unordered WRP

	General	$k \in \mathcal{O}(\frac{\log n}{\log \log n})$
Feasibility	P	
Optimality	NPH,APX,FPT	P

Open Questions

- Other special graph classes, e.g.: bidirected planar graphs
- Feasibility hardness for the ordered variant (we gave the optimality hardness)