Working Set Theorems for Routing in Self-Adjusting Skip List Networks

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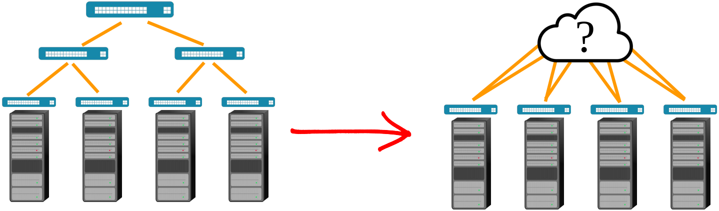
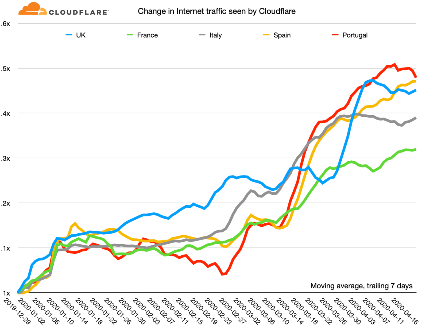
In a nutshell

Context: Self-Adjusting Networks

Goal: adjust a non-tree topology over unknown demand, minimize routing+adjustment costs



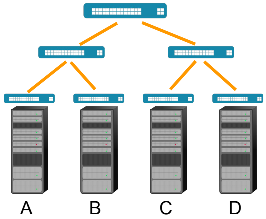
Data center traffic on the rise



- internet/data center traffic is increasing (even more in lockdowns!)

- packet switch bandwidth is increasing slower than the traffic increase rate!

A look inside: data center interconnects

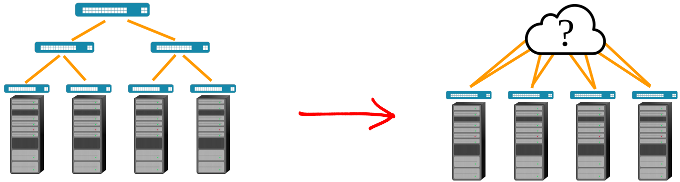


- data center top-of-the rack switch interconnects are currently **static**
- good design only for uniform demand patterns
- what if there is “elephant” traffic between (A, C) and (B, D)?

- Demand is skewed! [BAM10, GMP^{+}16]

- Need **dynamic** physical topologies!

Hardware support for dynamic connectivity

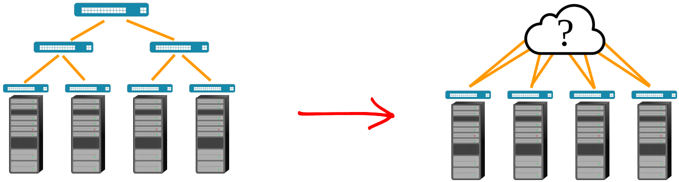


- dynamic physical topologies so far use: circuit switches, 60 GHz wireless, and **free-space optics** [GMP^{+}16]

- large number of topologies are possible (high maximum degree), low reconfiguration time

How should topology **adjust** over time to better serve the demand?

Emergence of Self-adjusting Networks (SANs)



Challenges:

1. **How** should topology change upon serving a request?

2. Is it possible to support **non tree-based** topologies?
   existing work focuses on tree-based topologies, e.g. SplayNet [SAS$^+$16]

3. What are the **performance** guarantees?

Roadmap

1. Model
2. Self-Adjusting Skip List Networks
3. Proving working set property
4. Concurrent requests

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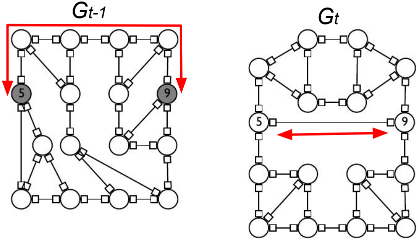
Abstracting SANs: input & algorithms

Input: G_0 an initial graph, $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_m)$ a sequence of communication requests

An **online SAN algorithm** $\mathcal{A}$ takes input G_0 and upon $\sigma_t = (s_t, d_t)$

- serves σ_t
- decides how to transform G_{t-1} to G_t

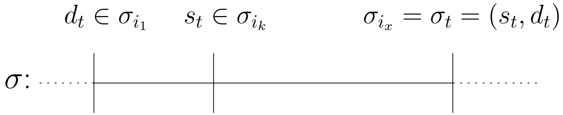


Based on [SAS+16, AS19]

SAN complexity

- $\mathit{cost}(\sigma_t)$: routing cost in G_{t-1} + cost of adjusting G_{t-1} to G_t

(pairwise) working bag $\mathit{WB}(\sigma_t)$: smallest subsequence ending in σ_{t-1} that contains both source and destination of σ_t



(pairwise) working set $\mathit{WS}(\sigma_t)$: distinct elements in working bag
(pairwise) working set number $|\mathit{WS}(\sigma_t)|$: size of working set

(pairwise) working set property: $\forall \sigma_t$: $\mathit{cost}(\sigma_t) = \mathcal{O}(\log |\mathit{WS}(\sigma_t)|)$

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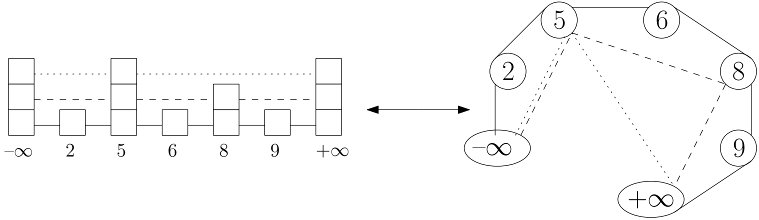
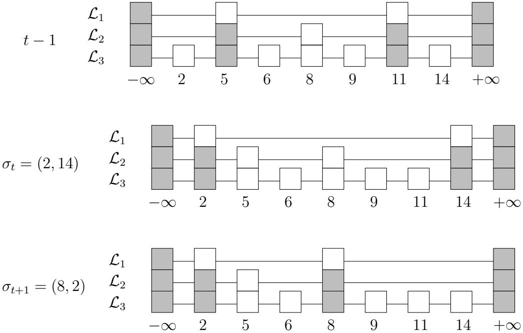
Skip List Networks



- element/link of the skip list = node/edge in graph (no duplicates)
- routing according to skip list finger search
- Good fit for networks due to: **local routing**, more resilient to link failures than trees, alternative to tree-based self-adjusting networks

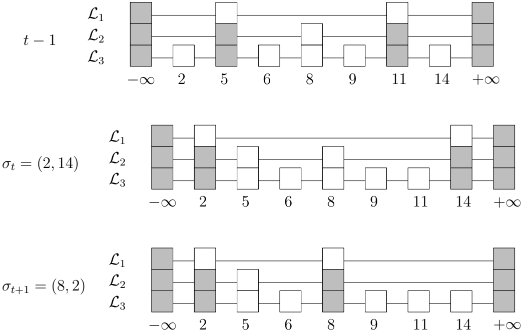
SASL^2: Self-Adjusting Skip List Network



- based on *SASL*, a statically optimal (for search sequences) self-adjusting skip list by Ciriani et al. [CFLM07]

- adjustment: promotion/demotion of nodes:
    - higher levels $\Longrightarrow$ shortest distance

SASL^2: Self-Adjusting Skip List Network



SASL^2: upon request (s, d): route (s, d), *adjust*(s), *adjust*(d)

- demoted nodes selected uniformly at random
- demotion is **graceful** and proportional to originating level

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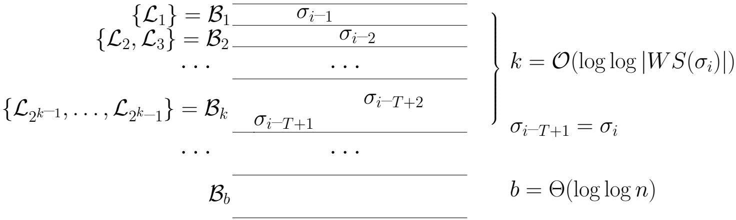
SASL^2 has the working set property

Step 1: Working set property for *SASL*

Fix a **search** request σ_i

Consider working bag of size T: $(\sigma_{i-T+1} \dots, \sigma_i)$



$\{\mathcal{L}_1\} = \mathcal{B}_1$ σ_{i-1}

$\{\mathcal{L}_2, \mathcal{L}_3\} = \mathcal{B}_2$ σ_{i-2}

$\dots$ $\dots$

$\{\mathcal{L}_{2^{k-1}}, \dots, \mathcal{L}_{2^k-1}\} = \mathcal{B}_k$ σ_{i-T+2} σ_{i-T+1}

$\dots$ $\dots$

$\mathcal{B}_b$

$k = \mathcal{O}(\log\log|WS(\sigma_i)|)$

$\sigma_{i-T+1} = \sigma_i$

$b = \Theta(\log\log n)$

[CFLM07]: items in working bag pushed down $\mathcal{O}(\log T)$ bands

This work: items in working bag pushed down $\mathcal{O}(\log|WS(\sigma_i)|)$ bands

$\Longrightarrow$ *SASL* has the working set property!

SASL^2 has the working set property

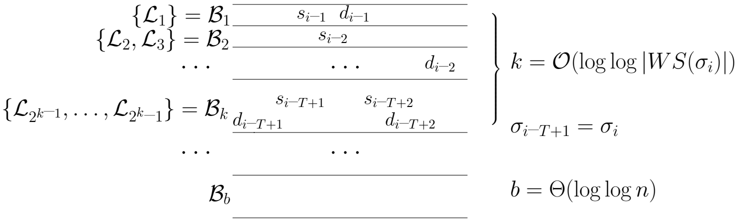
Step 2: Extending to *SASL*^{2}

Fix a communication request σ_i

Consider working bag of size T: $(\sigma_{i-T+1} \dots, \sigma_i)$

Step 2a: convert to sequence of search requests $(s_{i-T+1}, d_{i-T+1}, \dots, s_i, d_i)$, where $\sigma_t = (s_t, d_t)$

Step 2b: apply **pairwise** working set property definition!


$\{\mathcal{L}_1\} = \mathcal{B}_1$
s_{i-1} d_{i-1}
$\{\mathcal{L}_2, \mathcal{L}_3\} = \mathcal{B}_2$
s_{i-2}
$\dots$
$\dots$ d_{i-2}
$\{\mathcal{L}_{2^{k-1}}, \dots, \mathcal{L}_{2^k-1}\} = \mathcal{B}_k$
s_{i-T+1} s_{i-T+2}
d_{i-T+1} d_{i-T+2}
$\dots$ $\dots$
$\mathcal{B}_b$
$k = \mathcal{O}(\log\log|WS(\sigma_i)|)$
$\sigma_{i-T+1} = \sigma_i$
$b = \Theta(\log\log n)$

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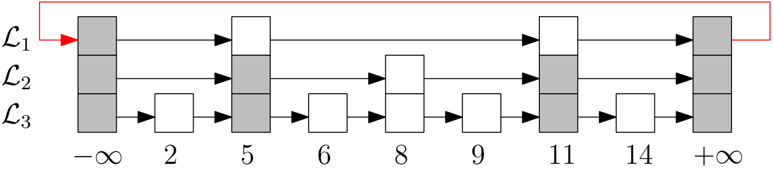
Serving requests concurrently



- Combine *SASL*^{2} with a **concurrent skip list implementation**, e.g. Herlihy et al. [HLLS07]

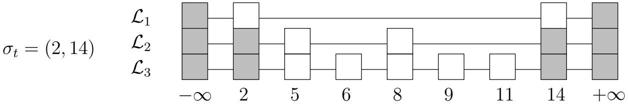
- Routing: use search routine (`findNode()`)

- Node promotion/demotion: use modified node add/delete routines

Wrap-up

- Existing/developing technology supports dynamic physical topologies [GMP+16]

- **Our contribution**: a self-adjusting skip list network with the (pairwise) working set property



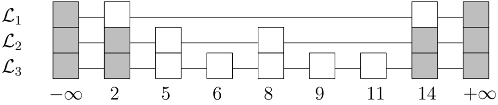
Future directions

- Lower bounds? (beyond the ones in SplayNet [SAS$^+$16])
- Extend other data structures to SANs

Thank you for your attention!

- Existing/developing technology supports dynamic physical topologies [GMP+16]
- **Our contribution**: a self-adjusting skip list network with the (pairwise) working set property

$\sigma_t = (2, 14)$



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