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Convergence of Even Simpler Robots without Position Information

Debasish Pattanayak, Kaushik Mondal,
Partha Sarathi Mandal
Indian Institute of Technology Guwahati, India
Stefan Schmid*
Aalborg University, Denmark & TU Berlin, Germany

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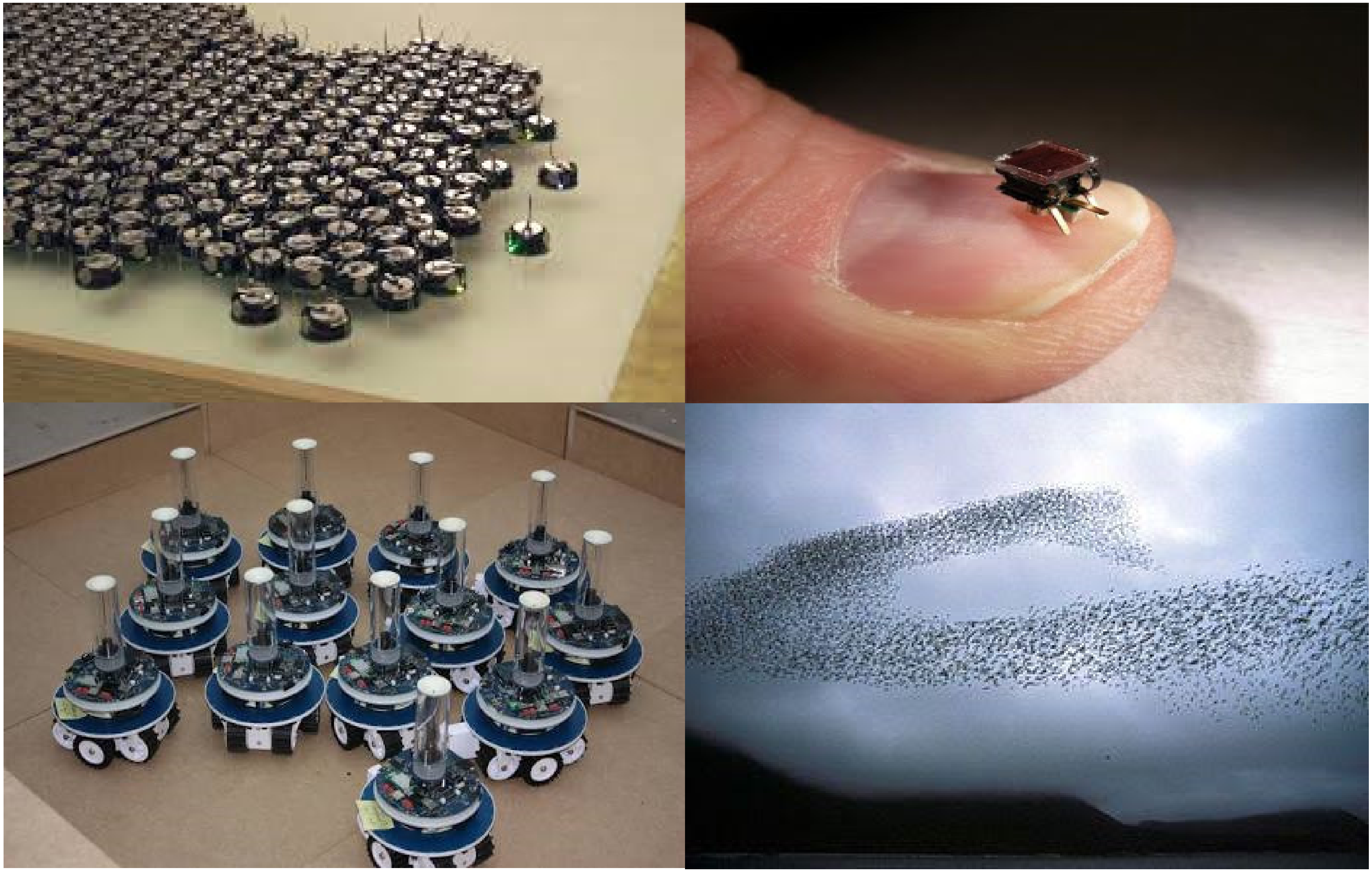


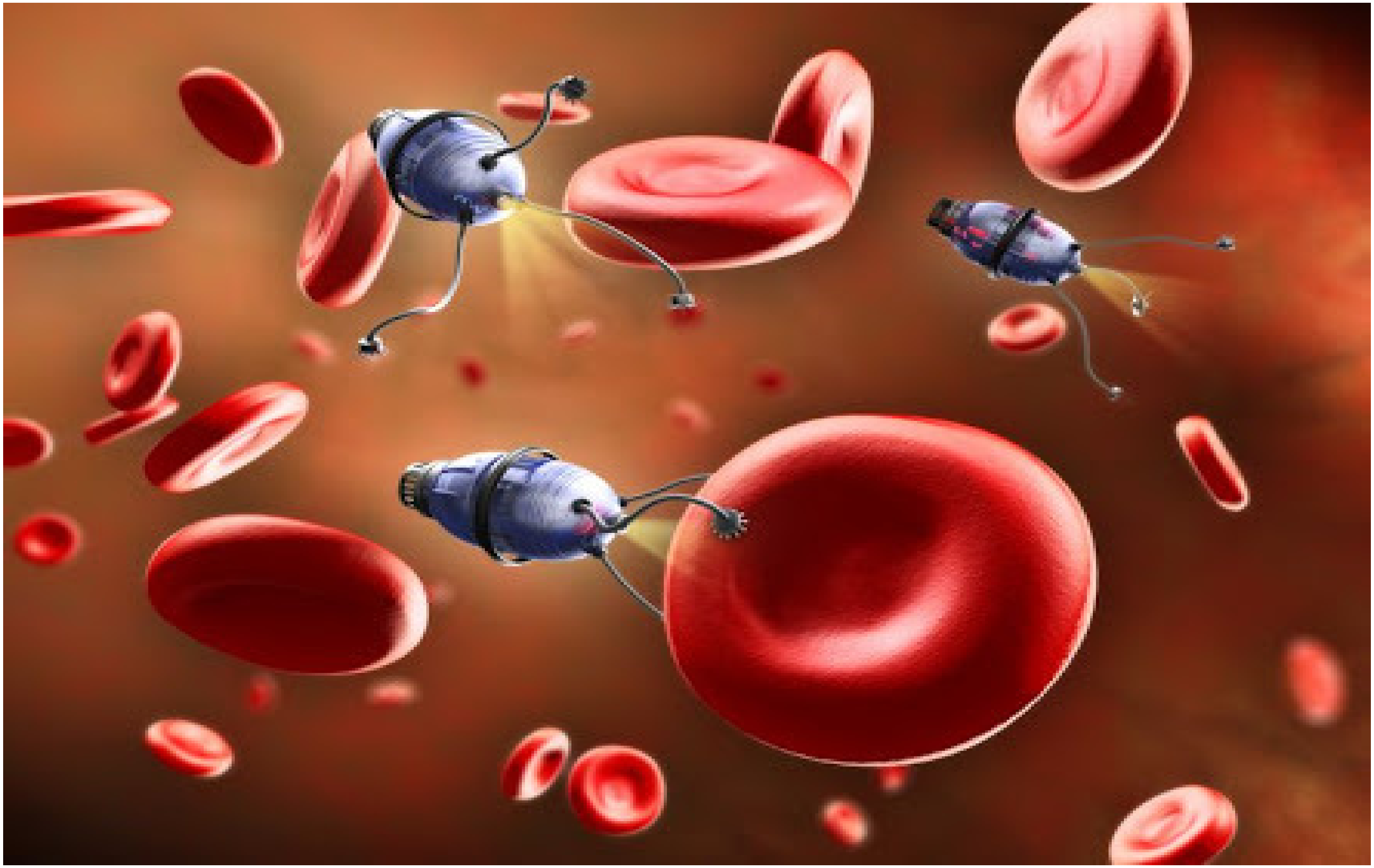
Image Source: EPFL, I-Swarm Project, Wikipedia

Swarm Robots

Image Source: EPFL

Applications

Disaster Rescue



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Applications

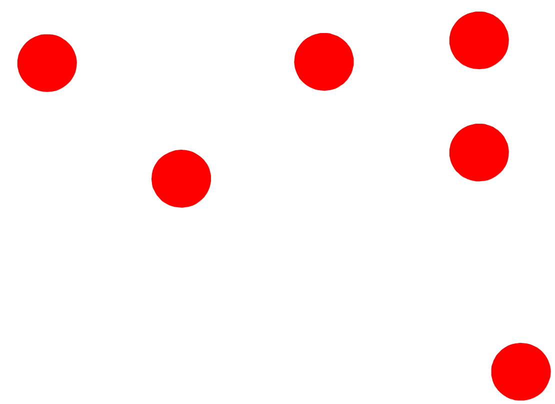
Nano-robots in blood stream

Outline

- Introduction to robots
- Computational model of robots
- Related works
- Monoculus robots
- Problem: Convergence
- Impossibility of convergence
- Convergence with
  - Locality Detection
  - Orthogonal Line Agreement
    - Termination requires memory
- Extension to d-dimension
- Simulation
- Conclusion & Future Works

Introduction to Robots

- Autonomous
- Homogeneous
- Anonymous
- Oblivious
- Silent
- Unlimited Visibility Range
- Point robots (collisions are ignored)



Computational Model

- States of Robot
  - *Look-Compute-Move*
- Common Knowledge
  - Axis-agreement
- Capability
  - Multiplicity Detection
- Scheduling Policy
  - Asynchronous (ASYNC)
  - Semi-synchronous (SSYNC).

General Problems

- Gathering: Robots have to gather at a non-predefined point.
- Pattern Formation: Robots have to form a given pattern.

Related Works

- Flocchini et al. [1] have introduced the notion of "weak robots" with following properties.
  - Autonomous
  - Anonymous
  - Oblivious
  - Silent
  - Axis-agreement
  - Multiplicity Detection
  - ASYNC scheduling.
  - Locate Position of other robots.
- They have investigated the common knowledge required to achieve Gathering and Pattern Formation with weak robots.

[1] Flocchini, P., Prencipe, G., Santoro, N., Widmayer, P.: Hard tasks for weak robots: the role of common knowledge in pattern formation by autonomous mobile robots. ISAAC 1999. LNCS, vol. 1741, pp. 93–102. Springer, Heidelberg (1999).

Related Works

- Cohen and Peleg [2] have pointed out these strong assumptions weak robots have
  - Can determine the position of other robots with completely accuracy.
  - The computations are precise.
  - It moves in a straight line towards the destination.

[2] Cohen, R., Peleg, D.: Convergence of autonomous mobile robots with inaccurate sensors and movements. SIAM J. Comput. 38(1), 276–302 (2008)

Related Works

- Cohen and Peleg [3] have proposed a center of gravity algorithm for convergence of two robots in ASYNC and any number of robots in SSYNC.
- Souissi et al. [4] have proposed an algorithm to gather robots with limited visibility if the compass achieves stability eventually in SSYNC.
- For two robots with unreliable compass Izumi et al. [5] have found that the limits of deviation angle ϕ to gather them in
  - SSYNC with $\phi < \frac{\pi}{2}$
  - ASYNC with $\phi < \frac{\pi}{4}$

[3] Cohen, R., Peleg, D.: Convergence properties of the gravitational algorithm in asynchronous robot systems. SIAM J. Comput. 34(6), 1516–1528 (2005)
[4] Souissi, S., D´efago, X., Yamashita, M.: Using eventually consistent compasses to gather memory-less mobile robots with limited visibility. TAAS 4(1), 9:1–9:27 (2009)
[5] Izumi, T., Souissi, S., Katayama, Y., Inuzuka, N., D´efago, X., Wada, K., Yamashita, M.: The gathering problem for two oblivious robots with unreliable compasses. SIAM J. Comput. 41(1), 26–46 (2012)

Our Contributions

- We initiate the study of a new kind of robot, the monoculus robot which cannot measure distances. The robot comes in two natural flavors
  - Locality Detection (L D)
  - Orthogonal Line Agreement (OLA)
- We present and formally analyze deterministic and self-stabilizing distributed convergence algorithms for both L D and OLA.
- We show our assumptions in LD and OLA are minimal in the sense that robot convergence is not possible for monoculus robots.
- Performance of our algorithms through simulation is reported.
- Our approach is generalized to higher dimensions and, with a small extension, supports termination.

Monoculus Robot

We introduce Monoculus Robots with following properties.

- Cannot measure distances (No depth sensing).
- Non-transparent
- It moves a fixed distance *b* in one move step
- No axis-agreement
- No multiplicity detection

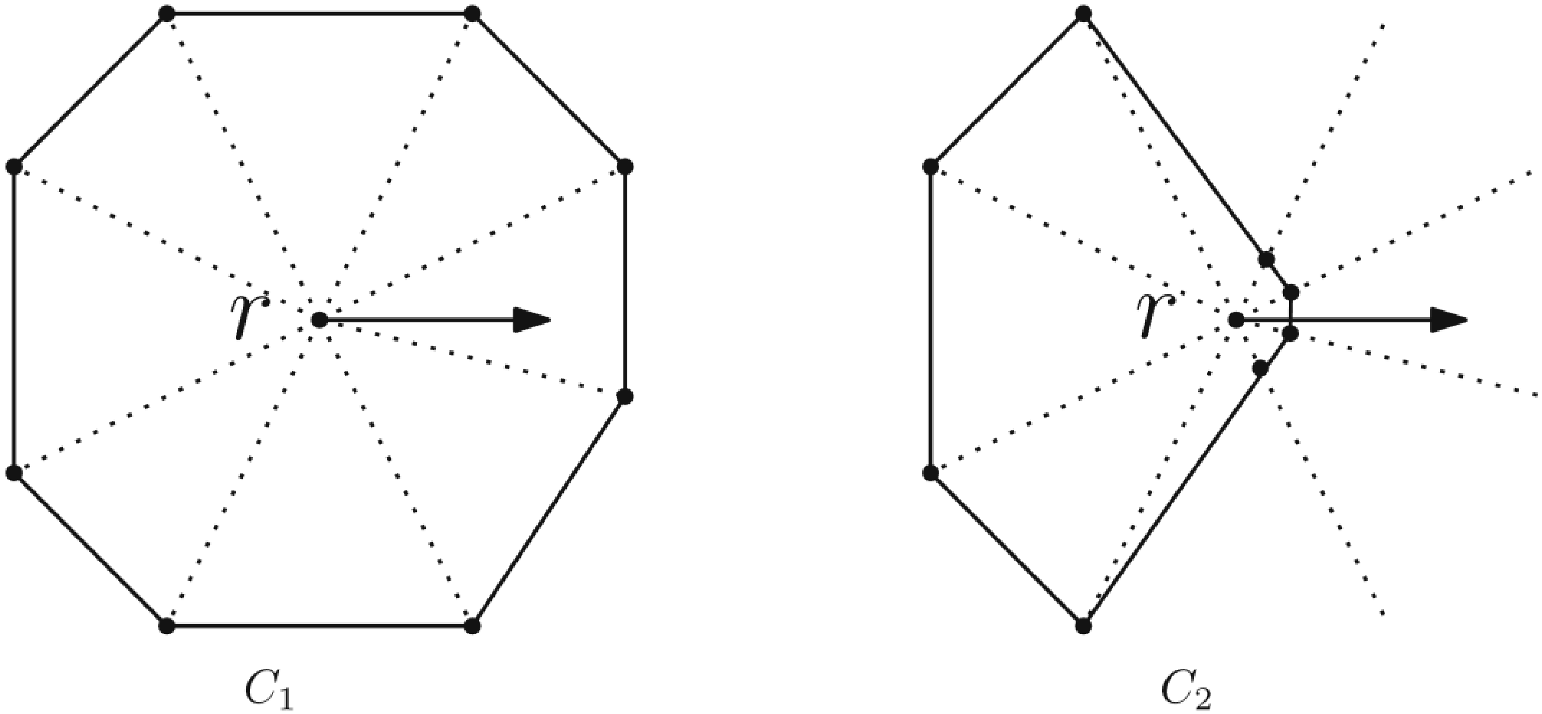
Convergence

- To gather in a small area whose position is not fixed beforehand.
- Achieved when the distance between any pair of robots is less than a predefined value ζ.
- The condition remains consistent subsequently.

Terminology

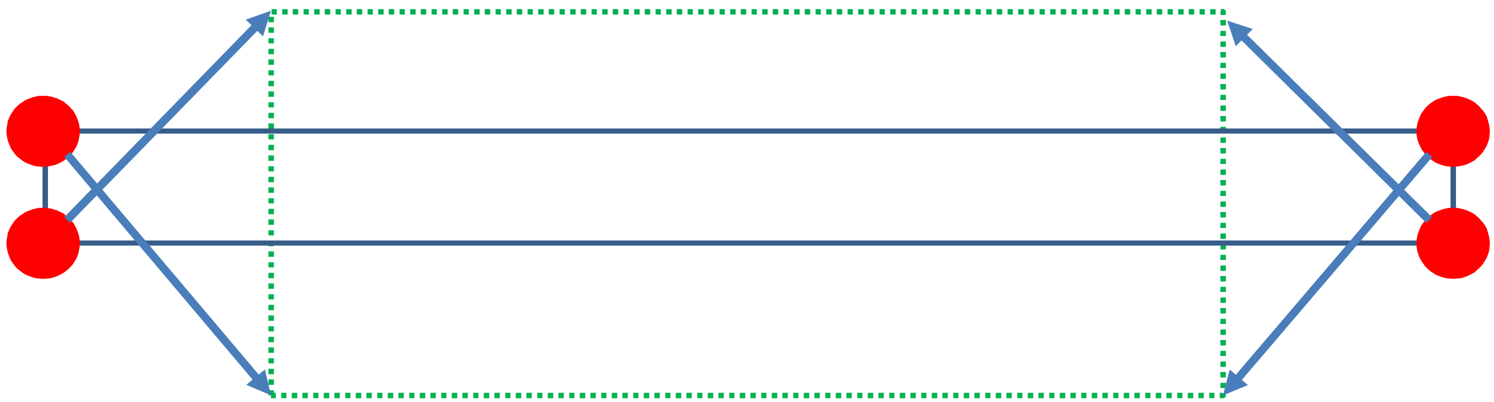
- The system of n robots are represented as
  $$R = \{r_1, r_2, \dots, r_n\}$$
- Observation of a robot,
  $$LC = \{\theta_1, \theta_2, \dots, \theta_k\}, k \leq n - 1$$
- Each $\theta \in LC$ is the angle another robot make in a robot's local coordinate system.
- A Configuration (C) is the set containing the position of robots.
- Convex Hull of a configuration at time t (C_t) is CH_t.

No deterministic convergence algorithm for monoculus robots



The configurations are indistinguishable from each other.

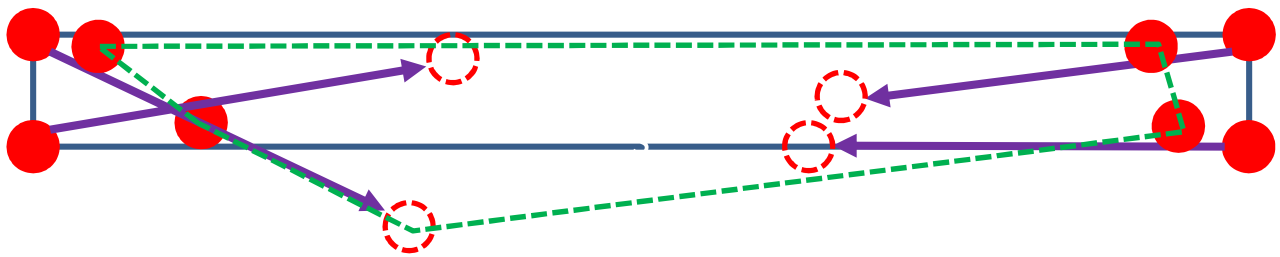
Non-monotonic Behaviour of Naïve Strategies



Going towards Angle Bisector

Boundary robots move along the angle bisector of the angle of convex hull

Non-monotonic Behaviour of Naïve Strategies



Going towards the median robot

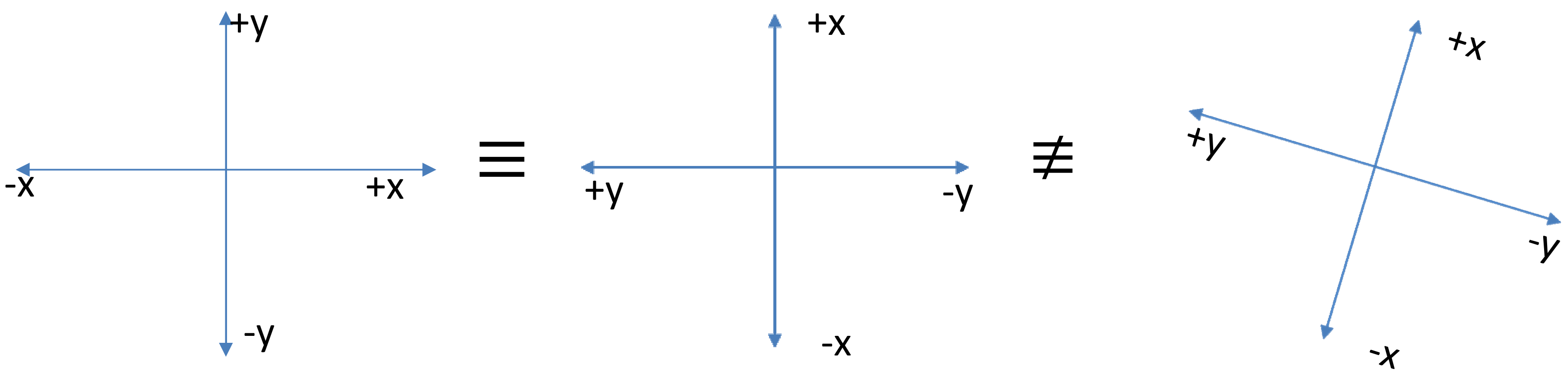
Boundary robots move towards the median robot in its local coordinate system

Locality Detection (LD) Model

- Determine whether its distance from any visible robot is greater than a predefined value c or not.
- Partition the set into two disjoint sets
  - LC_{local} : All robots are within distance c
  - $LC_{non\text{-}local}$: All robots are outside distance c

Orthogonal Line Agreement (OLA) Model

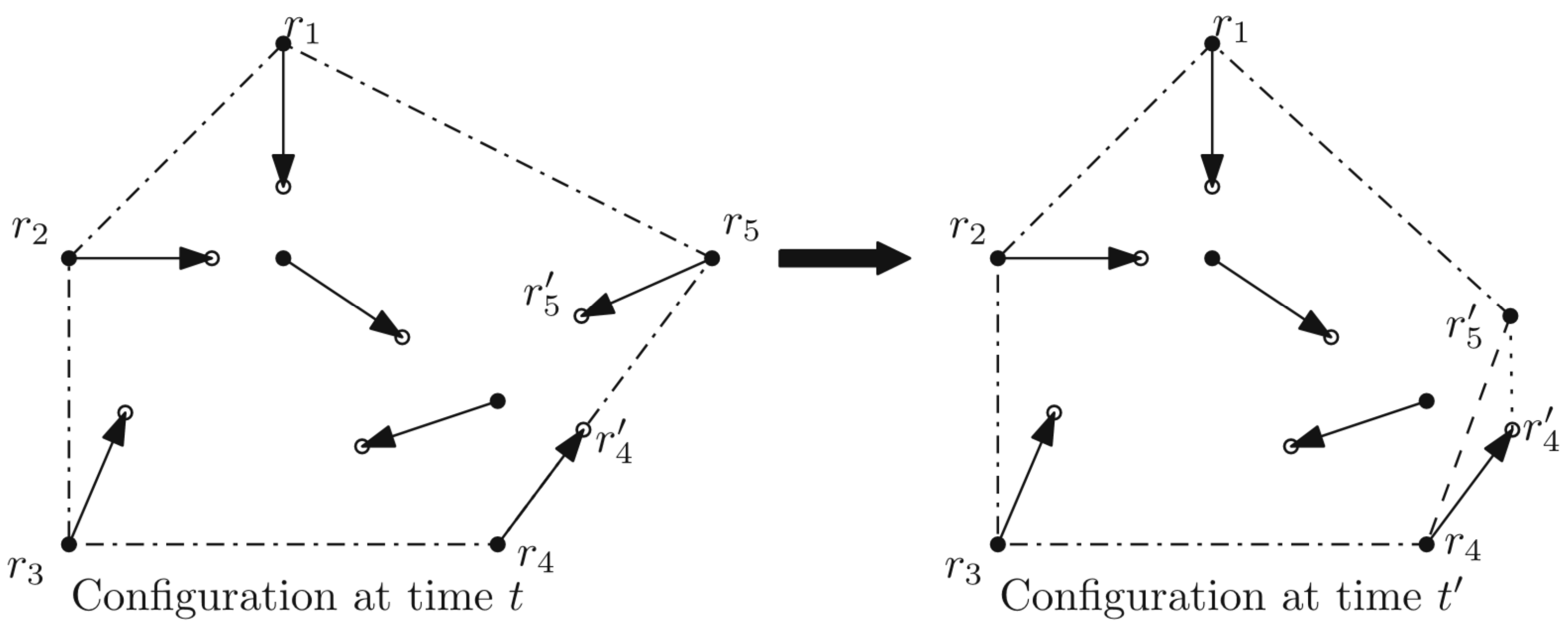
- Agree on a pair of orthogonal lines
  - No distinction between the lines is possible
  - No common sense of direction



Augmented Configuration

- The Augmented Configuration at time t (AC_t) is the configuration at time t (C_t) augmented with destinations of all the robots on or before time t.
- Convex Hull of the Augmented Configuration is the Augmented Convex Hull.

Augmented Convex Hull (ACH)



- r_4 computes destination to r_4' on or before t.
- r_5 moves to r_5' before r_4 starts moving at t' (>t).
- The Augmented Convex Hull includes r_4' since r_4' was computed before t'.

Algorithm for Locality Detection (LD)

Algorithm 1. CONVERGELOCALITY

Input : Any arbitrary configuration LC

Output: A direction θ towards the robot moves

if $|LC| = 1$ **then** // boundary robots in linear configuration

  Move distance b in the direction θ, where $\theta \in LC$

else

  if $|LC_{non-local}| \geq 1$ **then**

    Move distance b towards any θ, where $\theta \in LC_{non-local}$

  else

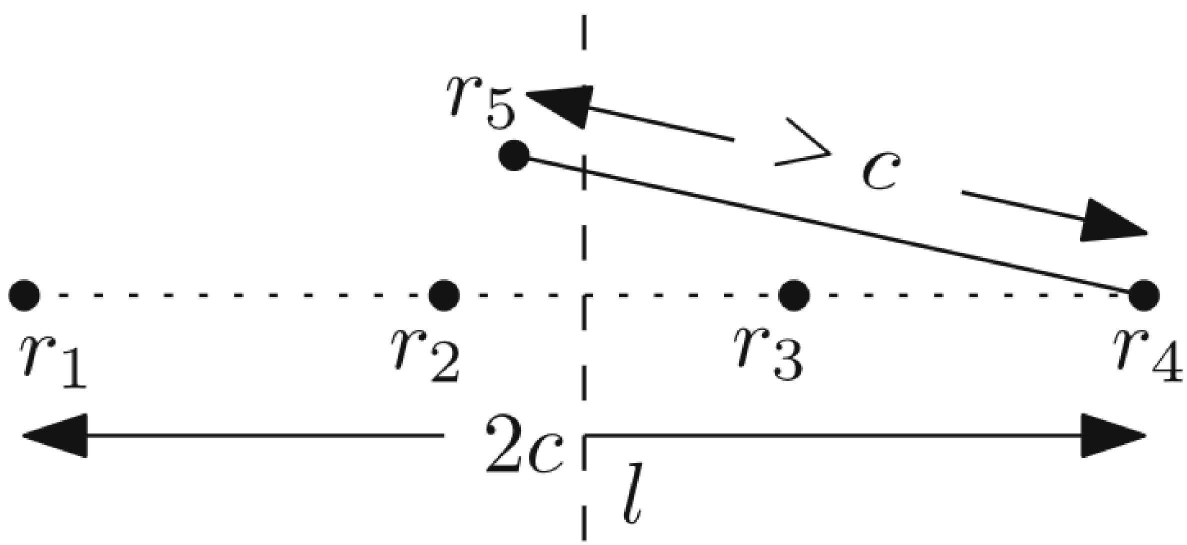
    Do not move `// All neighbor robots are within a distance` c

Linear Case

- The end robots move towards the only visible robot.

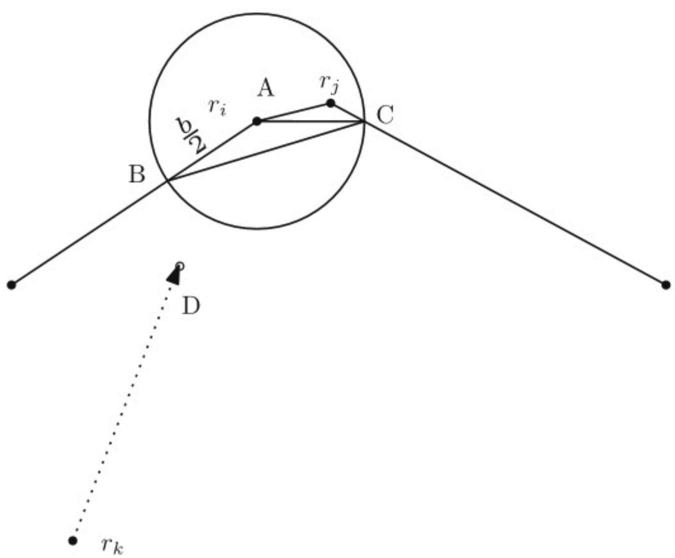
Convergence

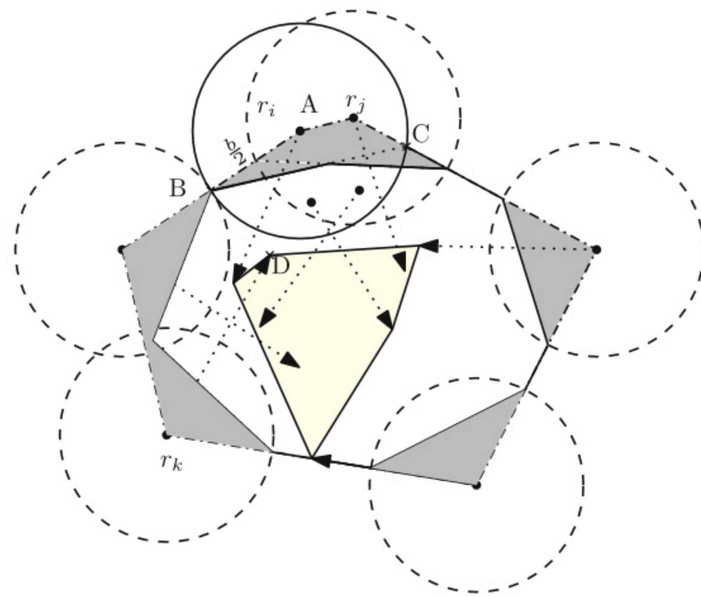
If there exists a pair of robots at distance more than 2c in a nonlinear configuration, then there exists a pair of neighbouring robots at distance more than c.



Convergence

For any time t' > t, before convergence, $ACH_{t'} \subseteq ACH_t$.





In the figure (right) the shadowed area is the decrement considered for each corner and the central convex hull inside solid lines is the new convex hull after every robot moves.

Decrement in Convex Hull

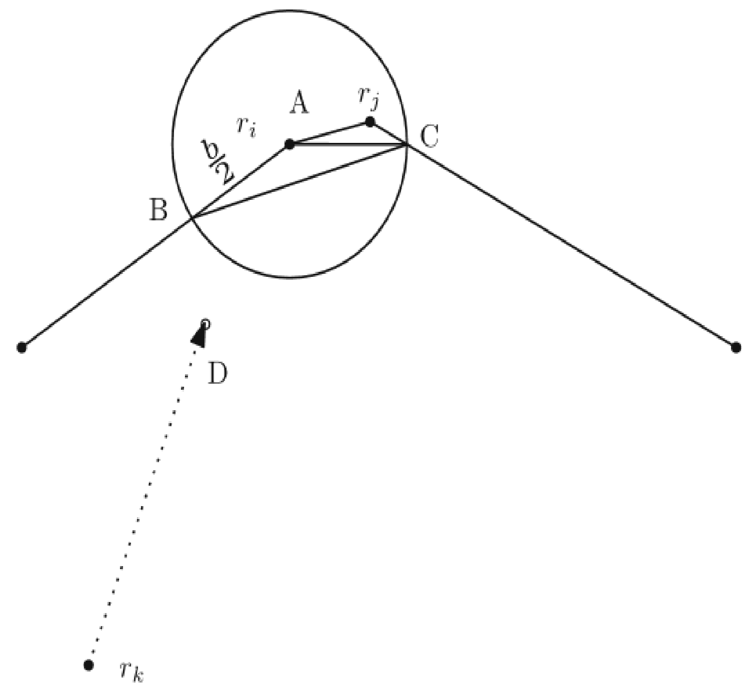
- There exist one angle in the Convex Hull in any configuration with an angle in some corner is less than

$$\left(1 - \frac{2}{n}\right)\pi$$

- The decrement is greater than

AB + AC – BC, i.e.,

$$b\delta = b\left(1 - \sqrt{\frac{1}{2}\left(1 + \cos\left(\frac{2\pi}{n}\right)\right)}\right)$$



Convergence and Complexity

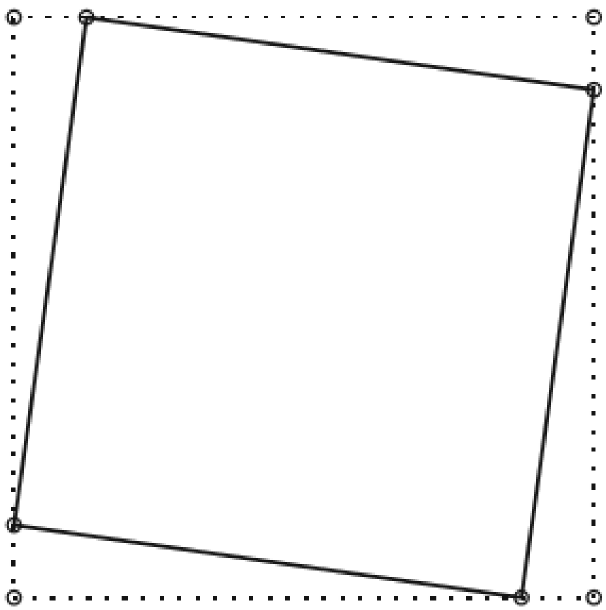
- The decrement is $b\delta$,

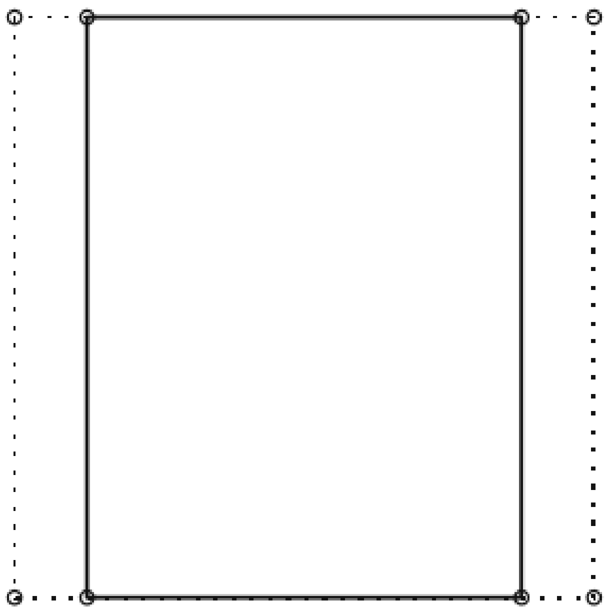
$$where\ \delta = 1 - \sqrt{\frac{1}{2}\left(1 + \cos\left(\frac{2\pi}{n}\right)\right)}$$

  is a constant.
- Perimeter of Convex Hull is smaller than $2\pi D$, where *D* is the diameter of smallest enclosing circle.
- Convergence constant $\zeta = 2c$
- Total time required is

$$\frac{\pi D - 2\pi c}{\delta b} = \Theta\left(\frac{D}{b}\right)$$

Remark: The decrement in Convex Hull

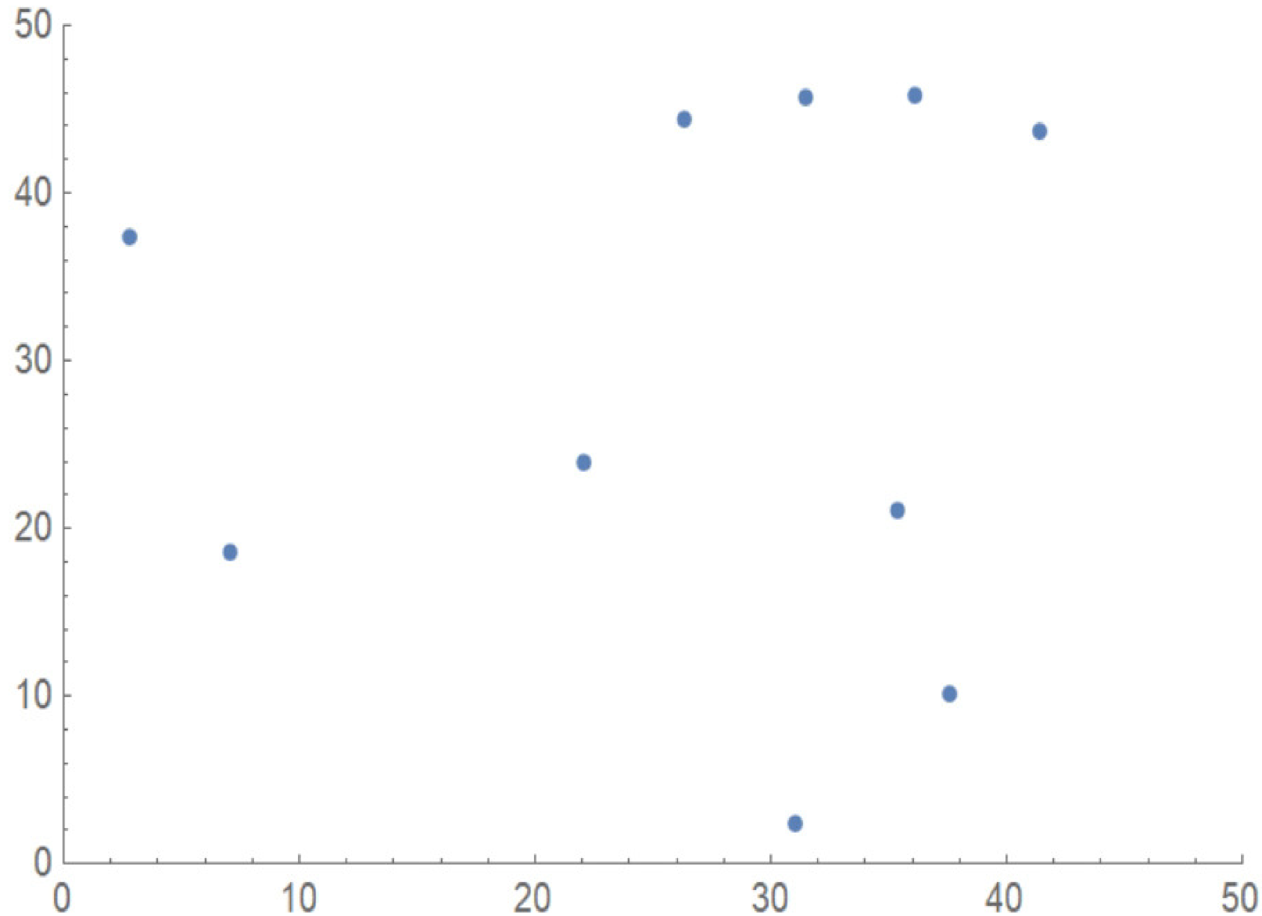




- The decrement happens even when all the robots move on the boundary.

ConvergeLocality Algorithm

for 10 Robots deployed in a square of side length 40



Algorithm for Orthogonal Line Agreement (OLA) Model

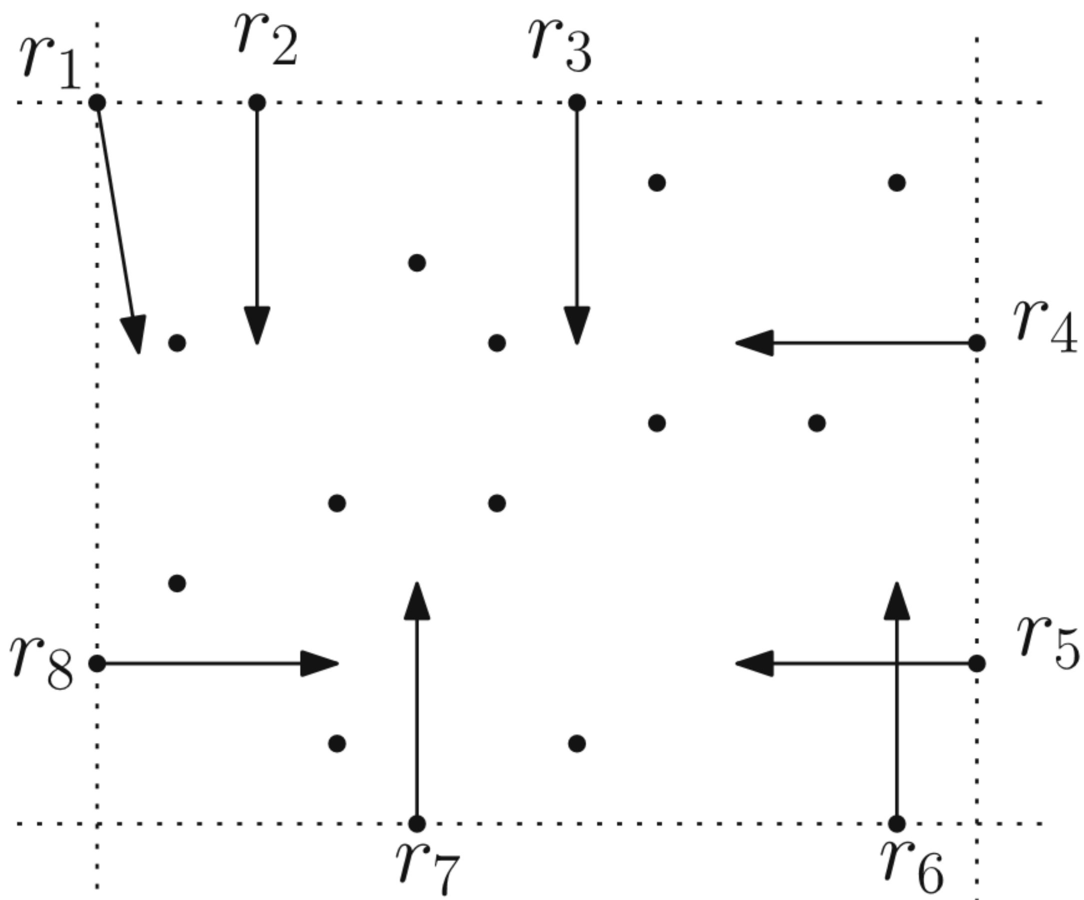
Algorithm 2. ConvergeQuadrant

Input : Any arbitrary configuration and robot r

Output: All robots are inside a square with side $2b$

```
if only one robot is visible then
    Move towards that robot
else if r is a boundary robot then
    Move perpendicular to the boundary to the side with robots
else if r is a corner robot then
    Move towards any robot in the non-empty quadrant
else
    Do not move                                    // It is an inside robot
```

Movement of Robots in OLA Model

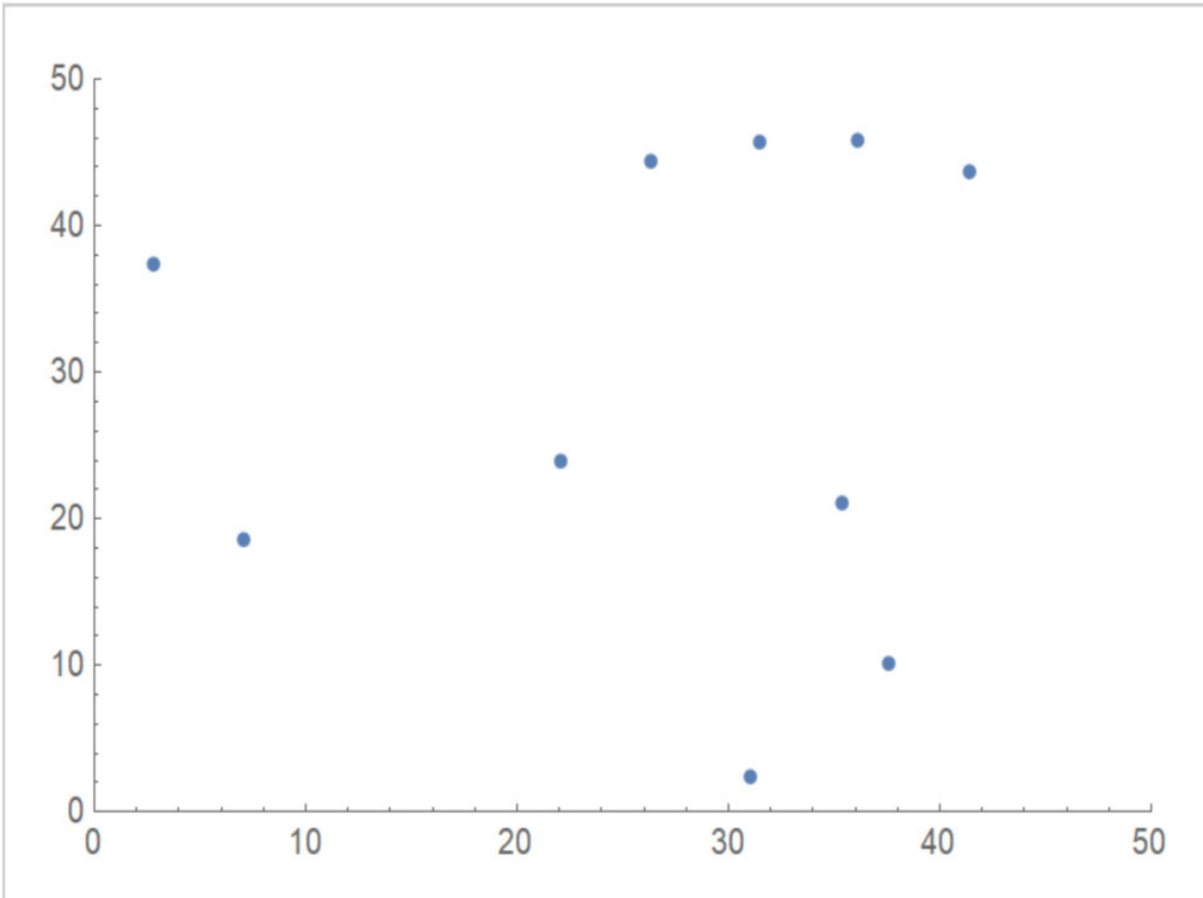


Convergence in OLA

- The distance between boundaries opposite to each other decreases monotonically over time.
- Once the distance between opposite boundaries becomes less than *2b*, it does not increase again.

ConvergeQuadrant Algorithm

for 10 Robots deployed in a square of side length 40



Termination using Memory

- Each robot contains two bits corresponding to the two extremes of each axis.
- A robot sets the bit to 1 if it ever finds itself in that particular extreme.
- A robot does not move if all the bits are set to 1.

Algorithm for OLA Model with Termination

Algorithm 3. CONVERGEQUADRANTTERMINATION

Input : Any arbitrary configuration and robot r with 4-bit memory
Output: All robots are inside a square with side $2b$

```
if the robot is on a boundary(ies) then
    set the corresponding bit(s) to 1
else
    Do nothing                                  // r is an inside robot
if r is a boundary robot and the bits corresponding to that dimension are not 1
  then
    Move perpendicular to the boundary to the side with robots
else if r is a corner robot then
    if Both bits corresponding to a dimension is 1 then
        Move in other dimension to the side with robots
    else
        Move towards any robot in the non-empty quadrant
else
    Do not move            // r is not on boundary OR all four bits are 1
```

Extension to *d*-Dimensions

- Both the algorithms can be extended to d-dimensions.
- In the LD model, similar argument can be used to prove convergence with a d-dimensional convex hull.
- For OLA model, we can have d perpendicular lines which agree with each other.
- Convergence in OLA for d-dimensions can be achieved with convergence in each of the dimensions.

Simulation

- Simulation parameters are set to be
  - $b = 1$
  - $c = 2$
  - Fully Synchronous Scheduling

Simulation

- $Centroid\ \{\bar{x}, \bar{y}\} = \left\{\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n}\right\}$
- Optimal Convergence Distance

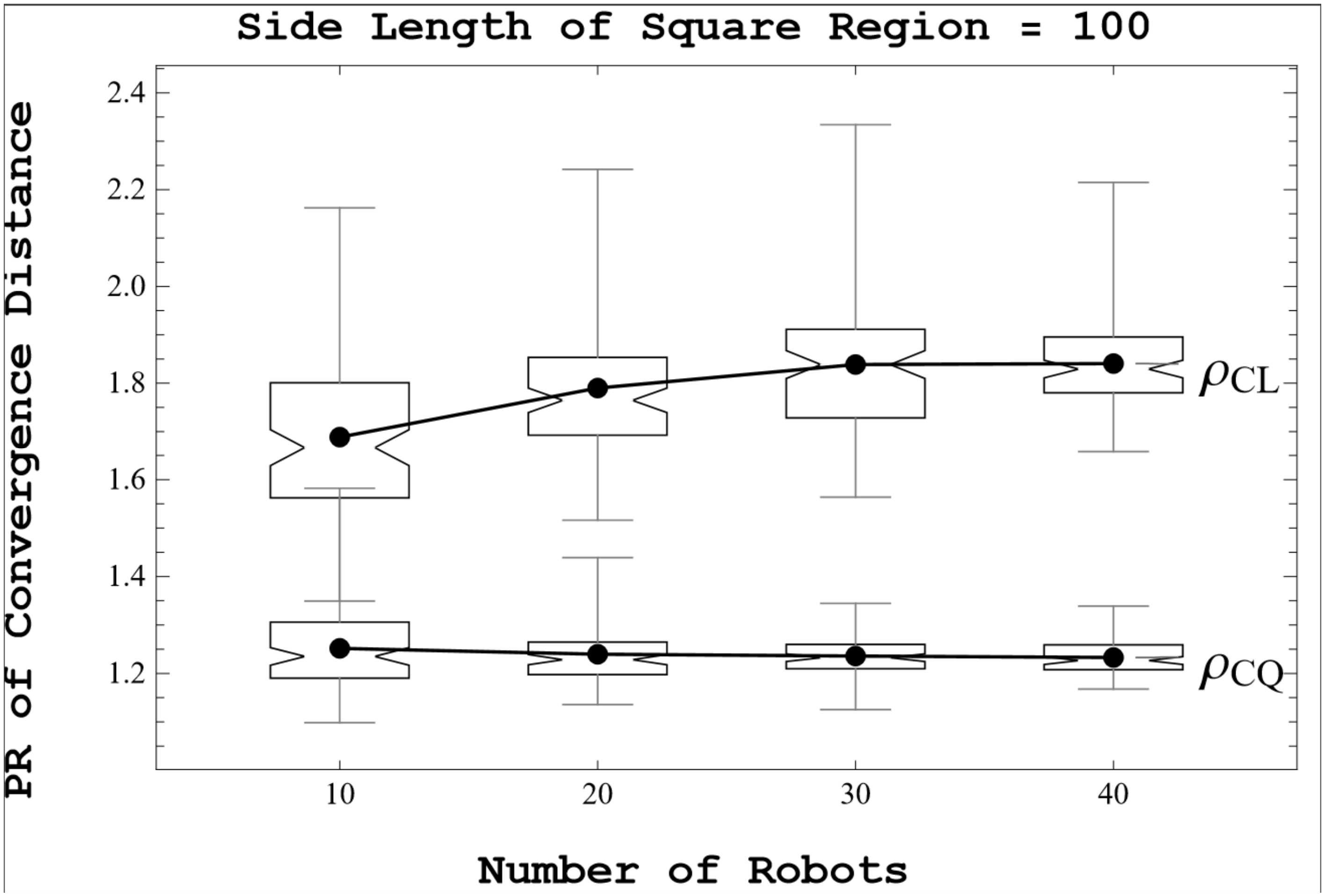
$$d_{opt} = \sum_{i=1}^{n} (d_i - 1), if\ d_i > 1$$

- Performance Ratio of Distance for CONVERGELOCALITY $\rho_{CL} = \frac{d_{CL}}{d_{opt}}$, where d_{CL} is the cumulative number of steps taken.
- Similarly ρ_{CQ} is defined for CONVERGEQUADRANT.

Simulation

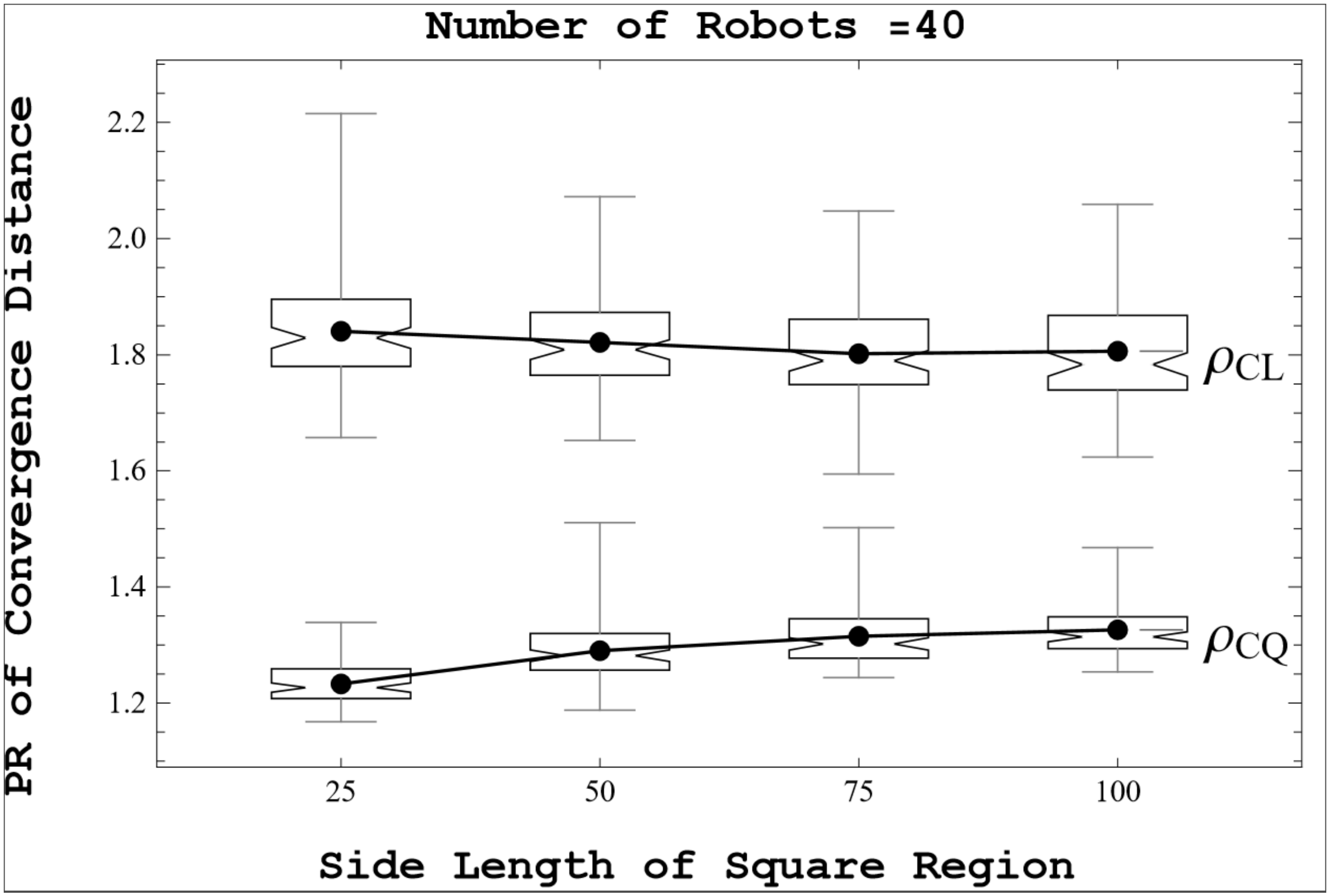
- d_{max} is the distance of farthest robot from centroid.
- t_{CL} is the total number of Synchronous rounds required by ConvergeLocality.
- Performace Ratio of Time for ConvergeLocality is $\tau_{CL} = \frac{t_{CL}}{d_{max}}$
- Similarly define τ_{CQ} for ConvergeQuadrant.

Simulation



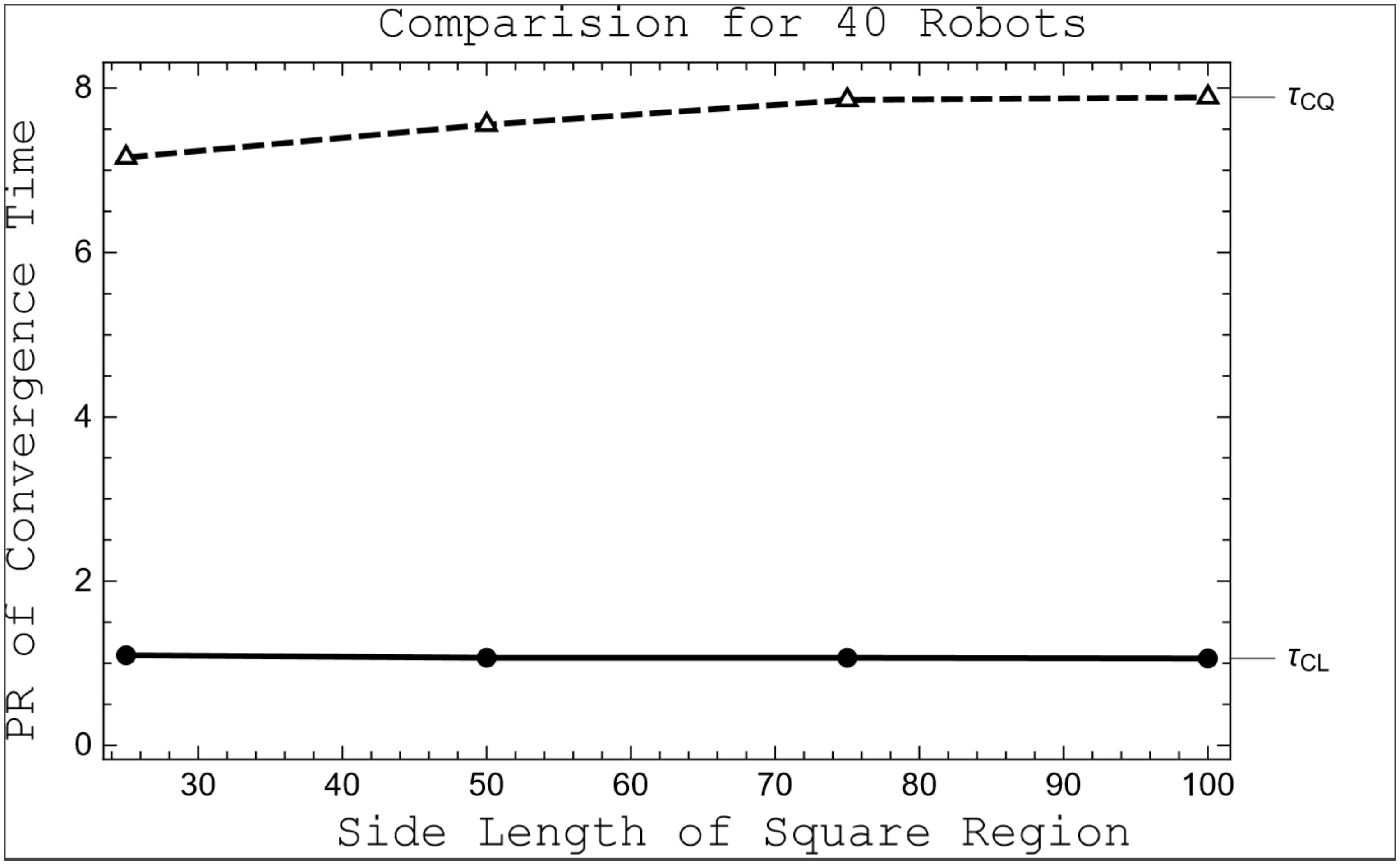
Side Length of Square Region = 100
PR of Convergence Distance
2.4
2.2
2.0
1.8
1.6
1.4
1.2
10
20
30
40
Number of Robots
ρ_{CL}
ρ_{CQ}

Simulation



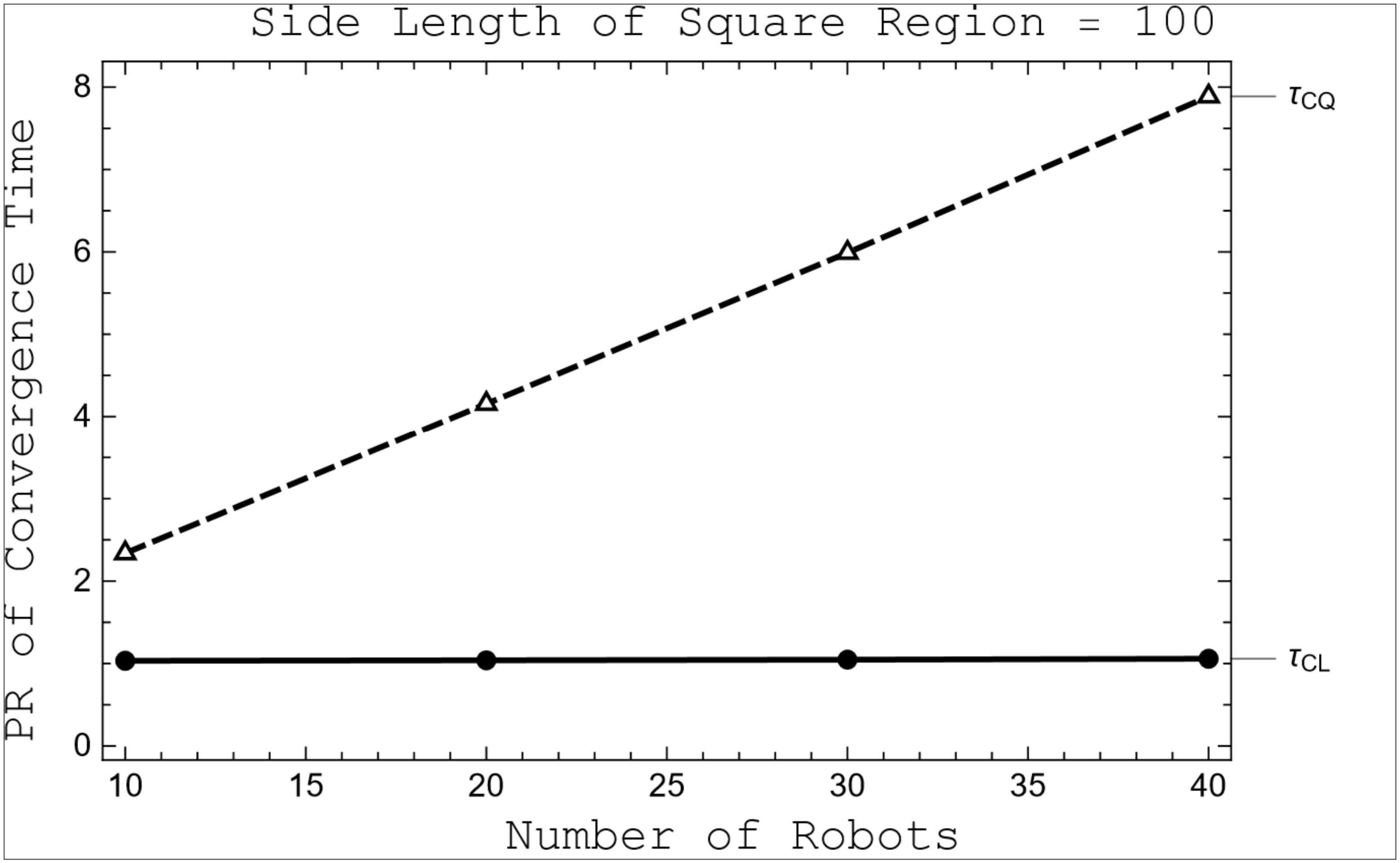
Number of Robots =40
PR of Convergence Distance
2.2
2.0
1.8
1.6
1.4
1.2
25
50
75
100
Side Length of Square Region
ρ_{CL}
ρ_{CQ}

Simulation



Comparision for 40 Robots
PR of Convergence Time
Side Length of Square Region
8
6
4
2
0
30
40
50
60
70
80
90
100
τ_{CQ}
τ_{CL}

Simulation



Side Length of Square Region = 100
PR of Convergence Time
Number of Robots
0
2
4
6
8
10
15
20
25
30
35
40
τ_{CQ}
τ_{CL}

Conclusion & Future Works

- We introduced monoculus robots concept.
- Proposed two basic models for convergence
  - Locality Detection (L D)
  - Orthogonal Line Agreement (OLA)
- We present and formally analyze deterministic and self-stabilizing distributed convergence algorithms for both LD and OLA.
- Proved that convergence is impossible with out these additional capabilities (LD or OLA)
- Regarding Future Works:
  - From simulations we have found that the Angle bisector and median strategies lead to successful convergence, but the proof remains a challenge.
  - It also remains to check whether the system is robust enough to tolerate errors in measurement.

Thank You!