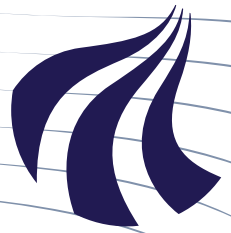
Self-Adjusting Linear Networks

with Chen Avin* and Stefan Schmid†

*Ben Gurion University, †University of Vienna

Ingo van Duijn

Department of Computer Science
Aalborg University
Denmark



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Baby Steps towards Self-Adjusting Linear Networks

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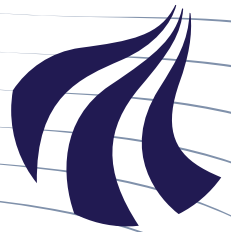

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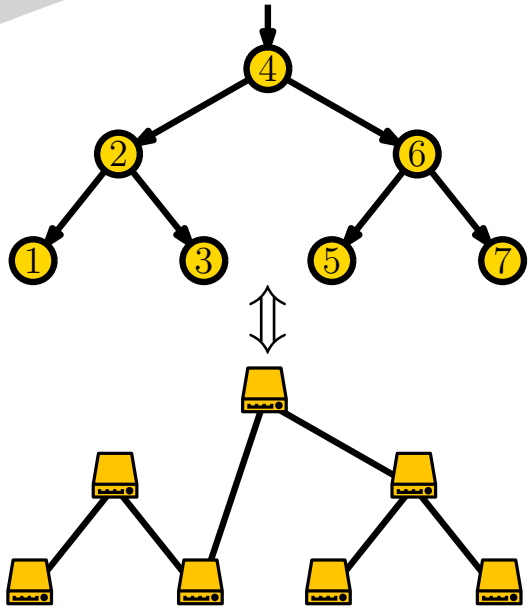
Overview

Overview

Intro: correspondence between

- Self-Adjusting Networks
- Dynamic Data structures



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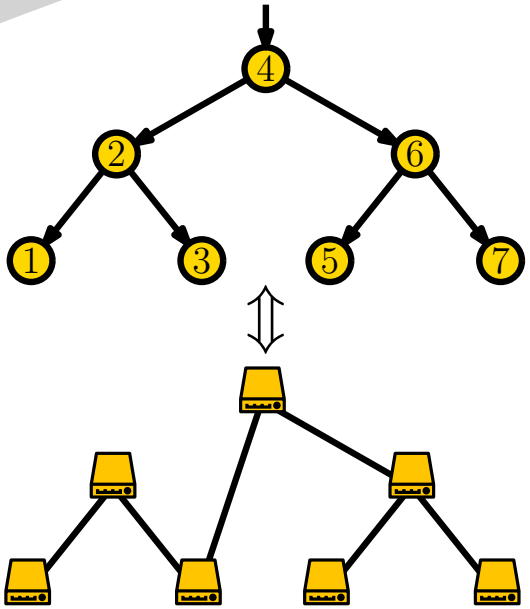
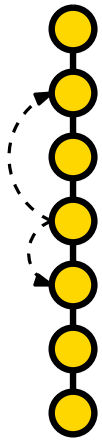
Case study:
- List Access
- List Communication





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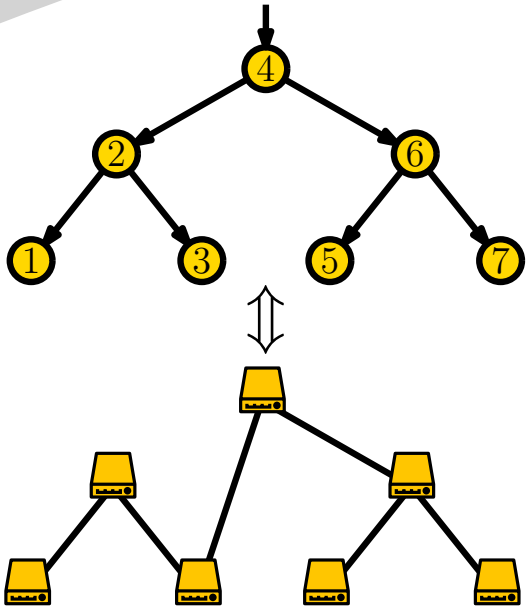
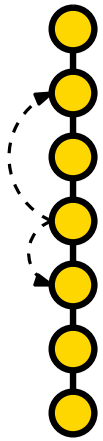
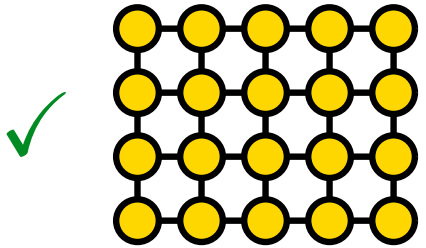
Results:
- Grid Network Bounds







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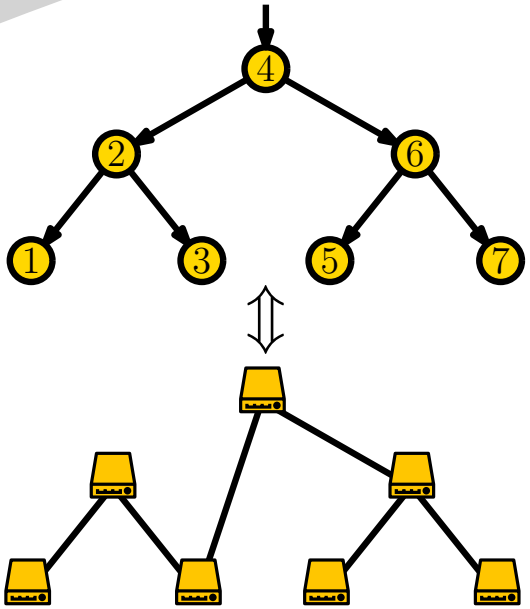
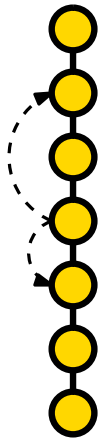
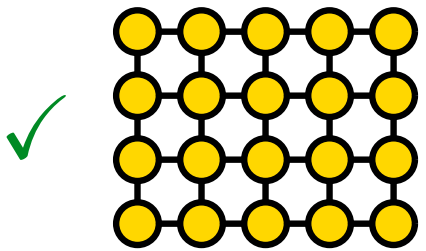
Lower Bound Proof Sketch







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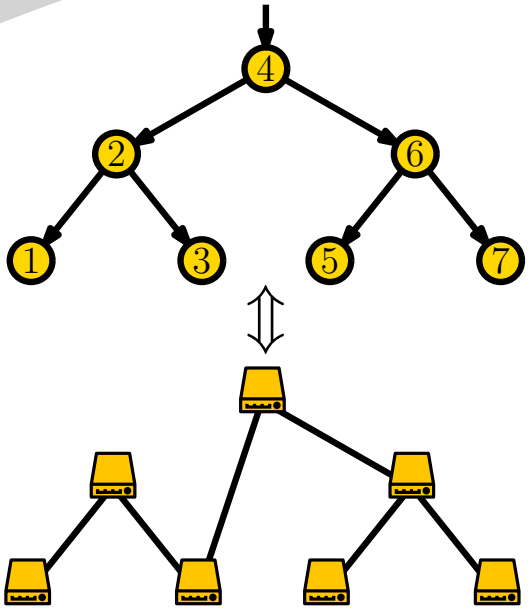
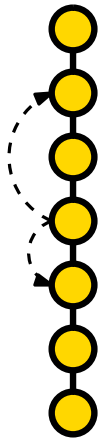
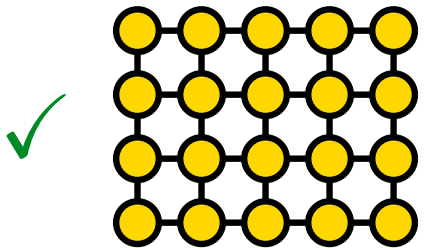
Future Work





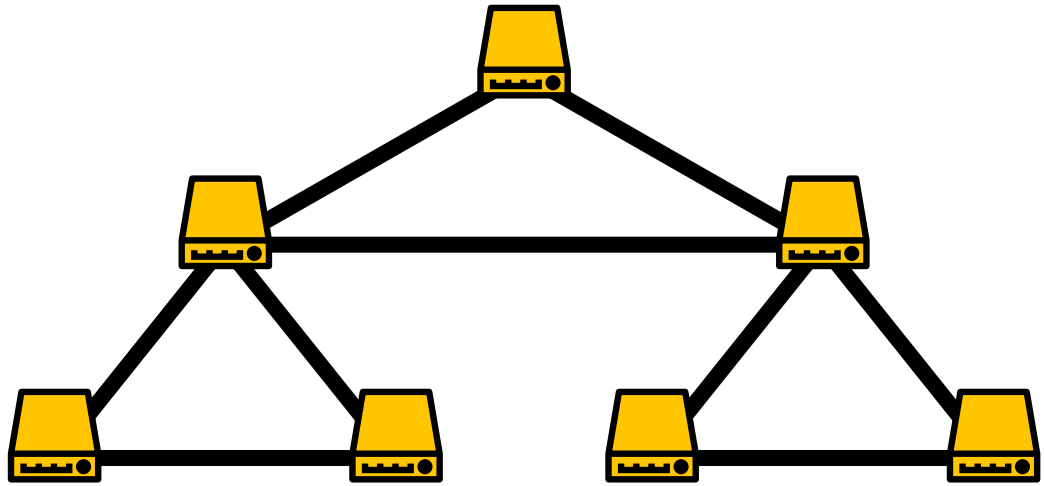


Self-Adjusting Networks

Self-Adjusting Networks

The Setting:

- Network



Self-Adjusting Networks

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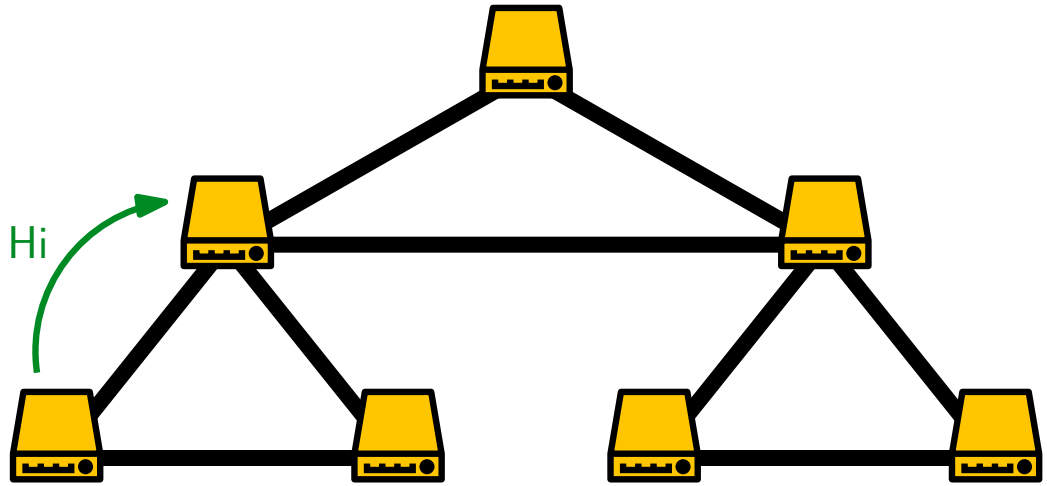

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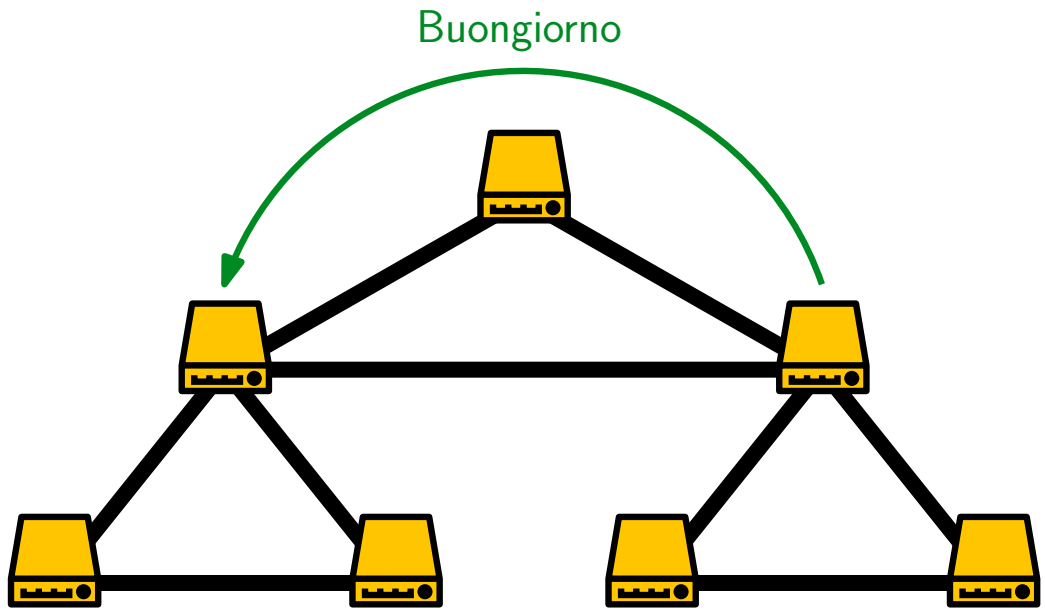

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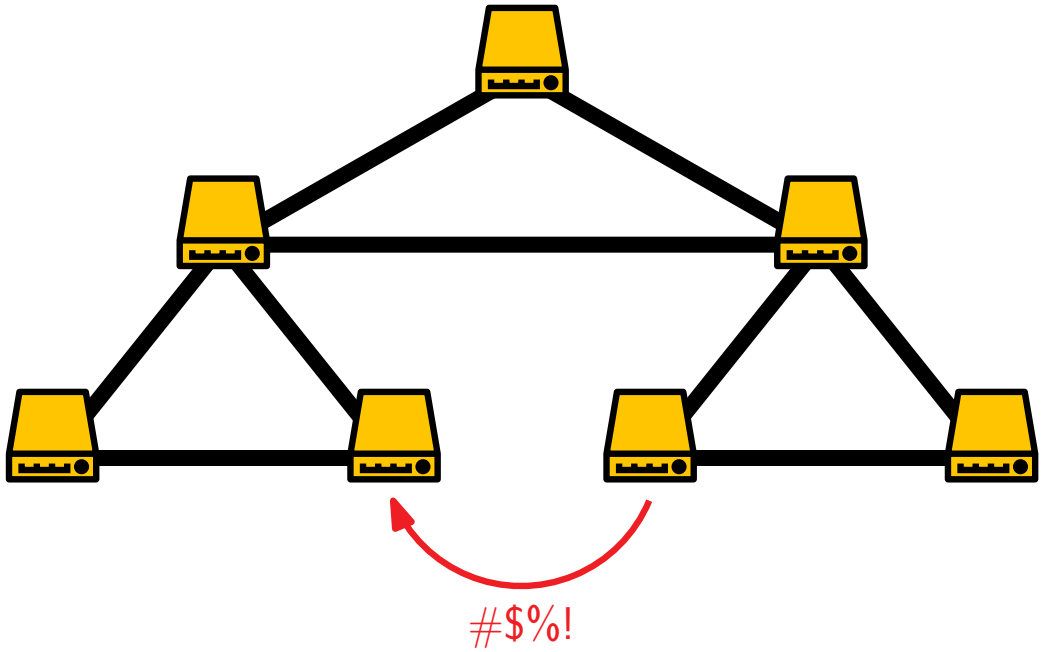
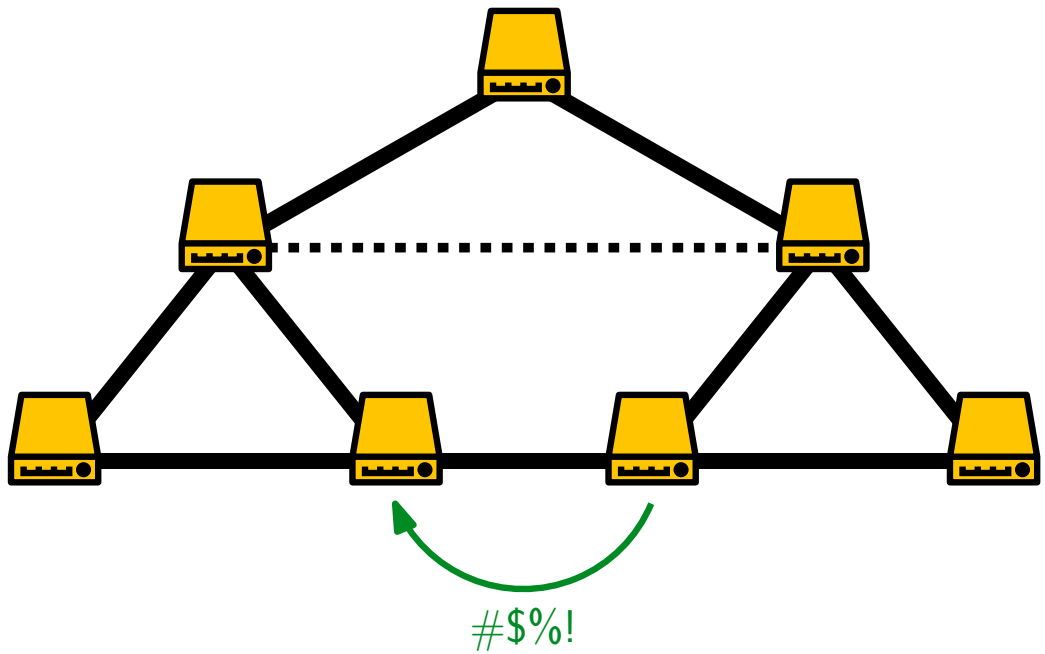


Self-Adjusting Networks

The Setting:

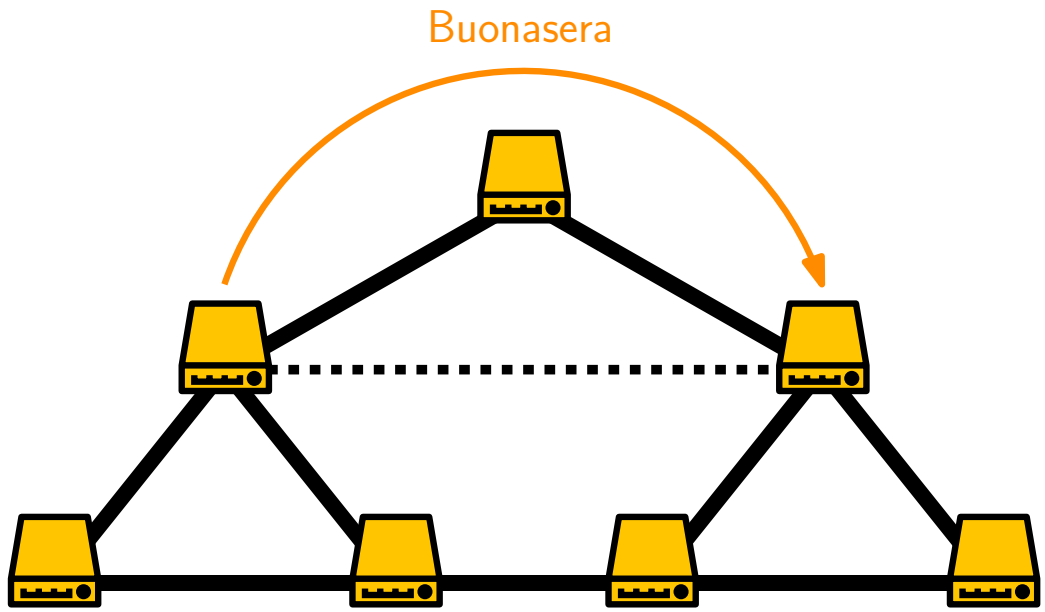
- Network
- Communication
- Adjustments



Self-Adjusting Networks

The Setting:

- Network
- Communication
- Adjustments
- Online



Self-Adjusting Networks

The Setting:

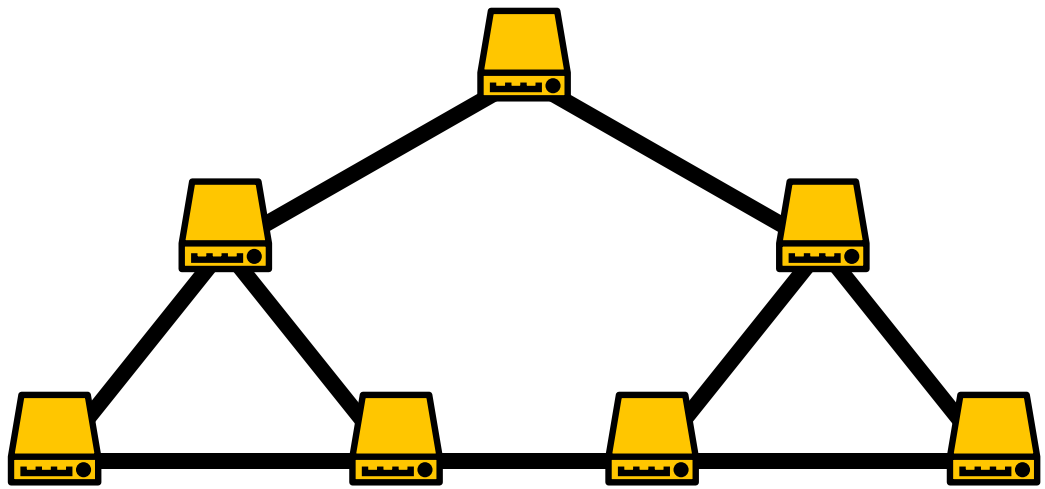
- Network
- Communication
- Adjustments

} Cost!

- Online

The Goal:

- Minimise Cost



Self-Adjusting Networks

The Setting:

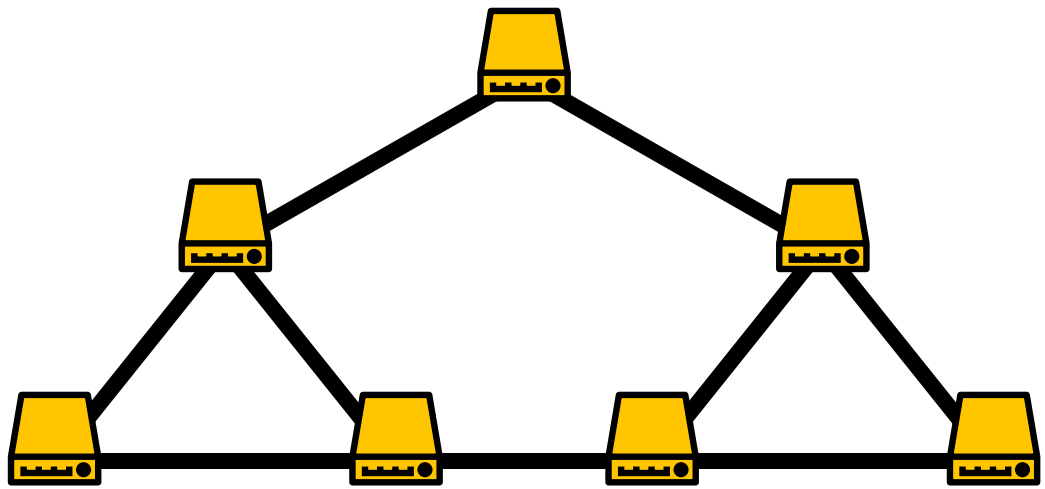
- Network
- Communication
- Adjustments

} Cost!

- Online

The Goal:

- Minimise Cost
- (Distributed Algorithm)



Self-Adjusting Networks

The Setting:

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} Cost!

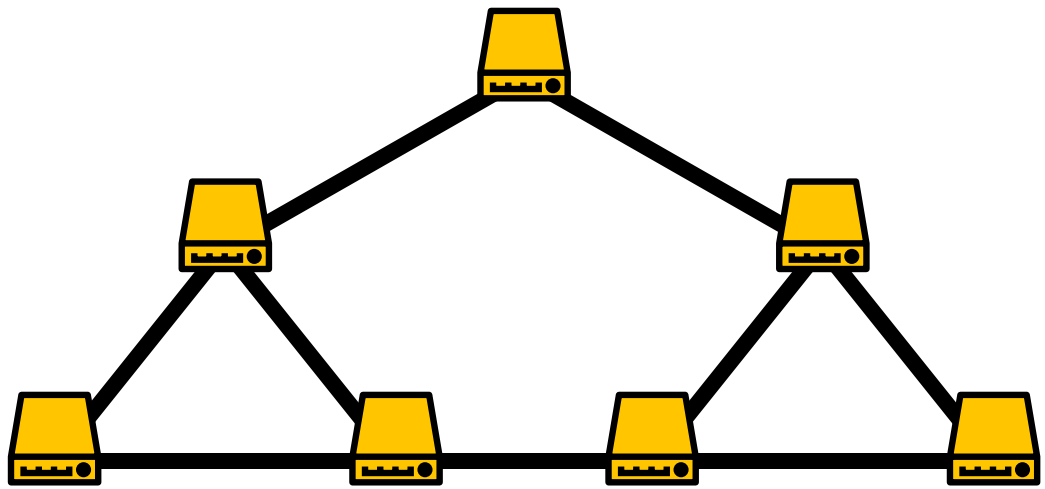
- Online

The Goal:

- Minimise Cost
- (Distributed Algorithm)

The Questions:

- Which Model?



Self-Adjusting Networks

The Setting:

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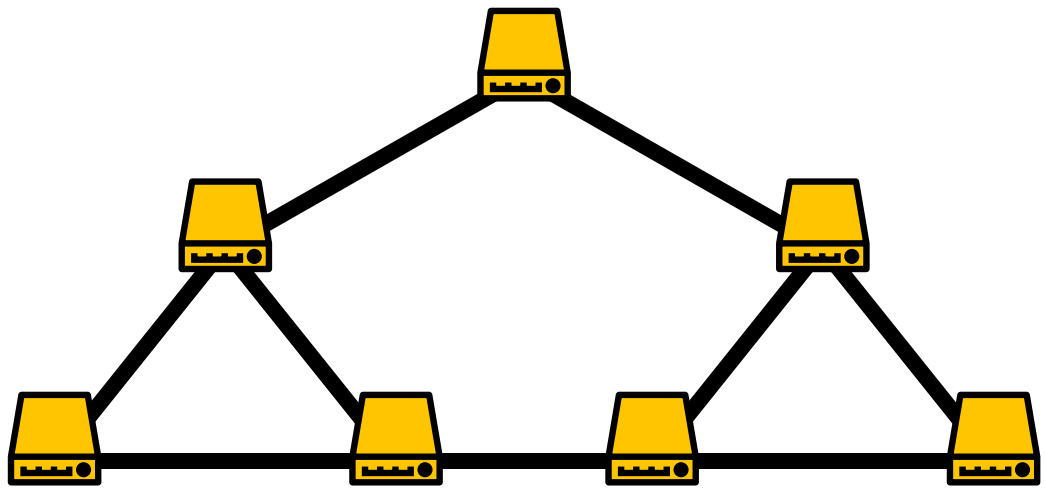
} Cost!

The Goal:

- Minimise Cost
- (Distributed Algorithm)

The Questions:

- Which Model?
- Formal Guarantees?

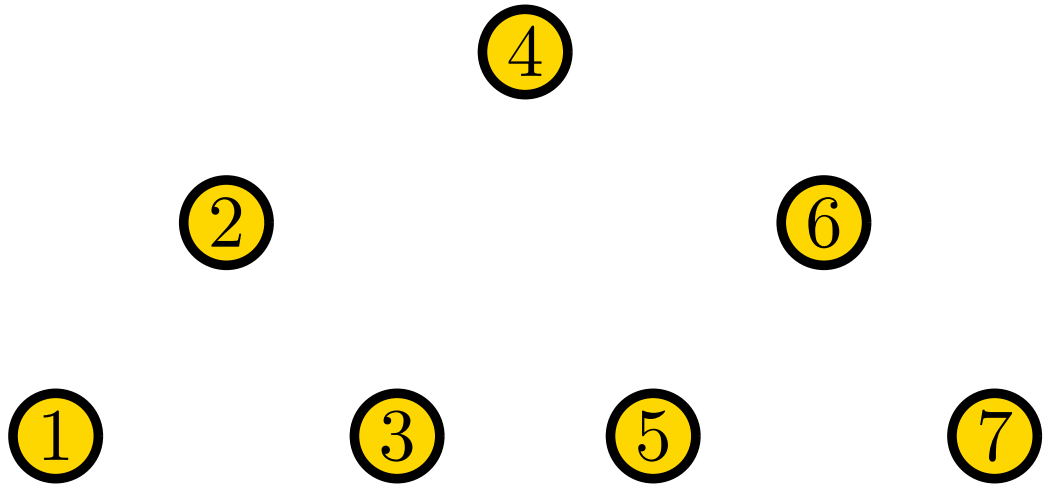


Dynamic Dictionaries

Dynamic Dictionaries

The Setting:

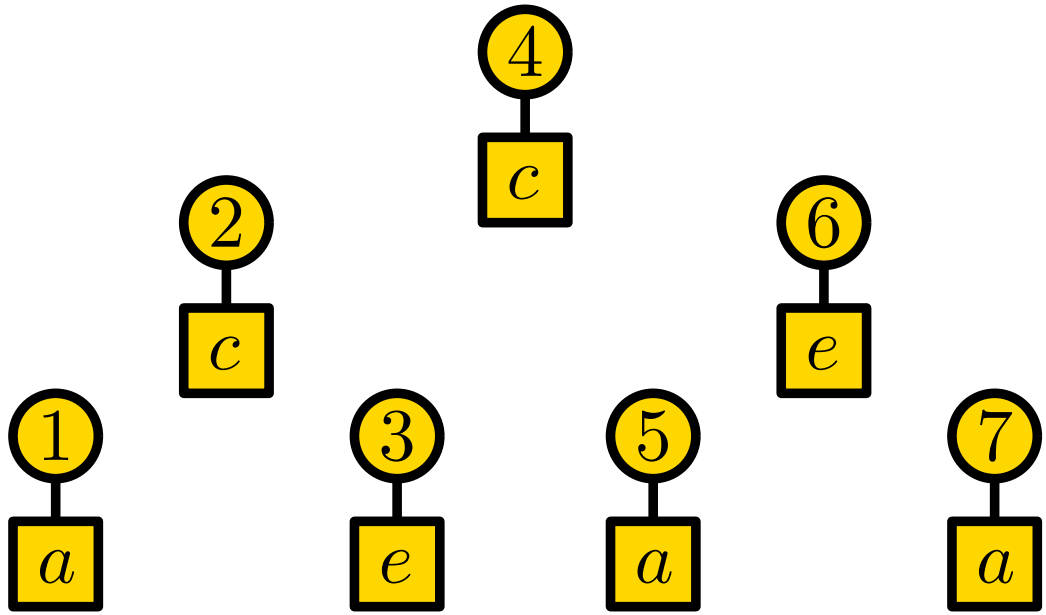
- Keys



Dynamic Dictionaries

The Setting:

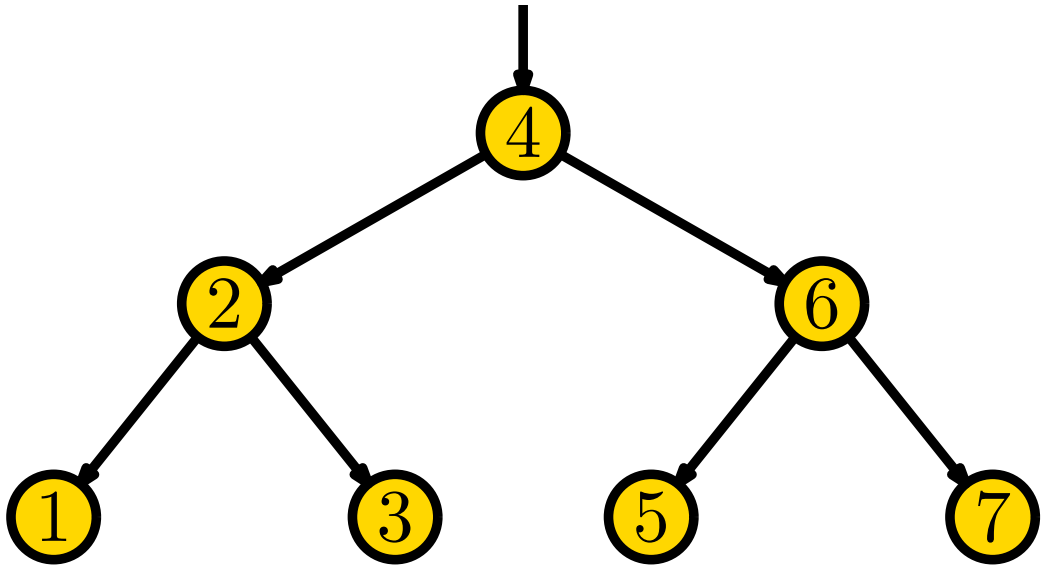
- Keys (with data)



Dynamic Dictionaries

The Setting:

- Keys
- Pointer Structure

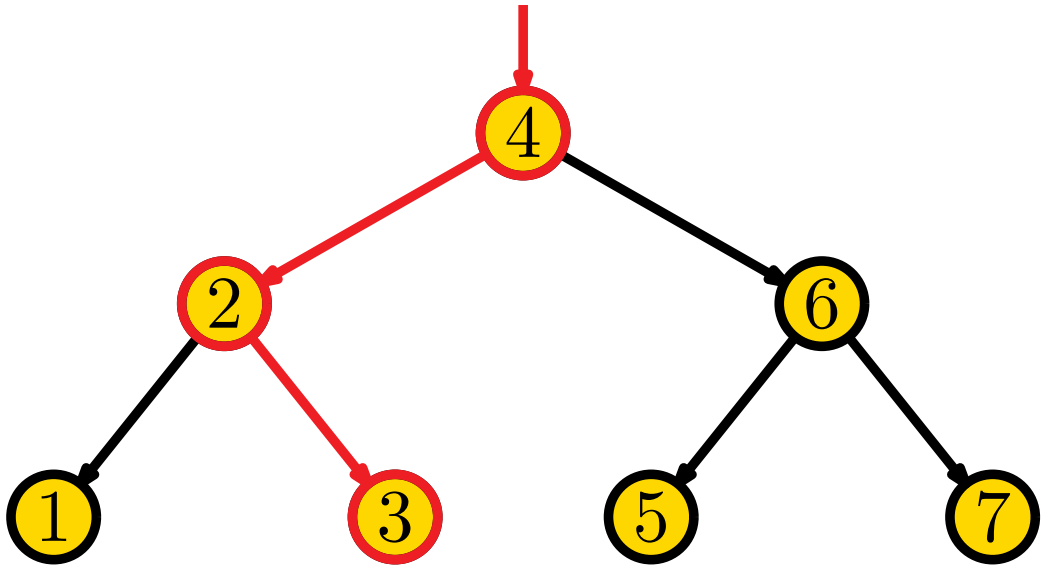


Dynamic Dictionaries

The Setting:

- Keys
- Pointer Structure
- Key Access

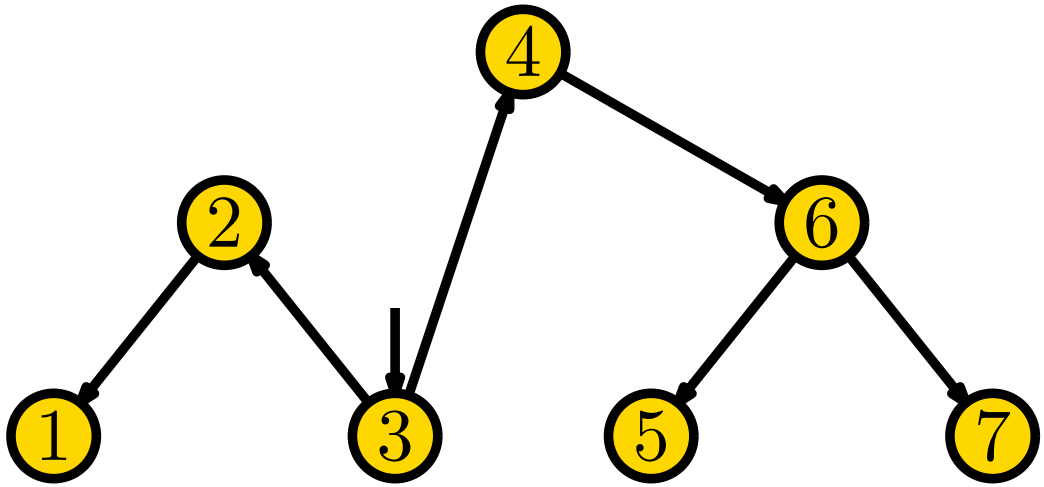
Access 3



Dynamic Dictionaries

The Setting:

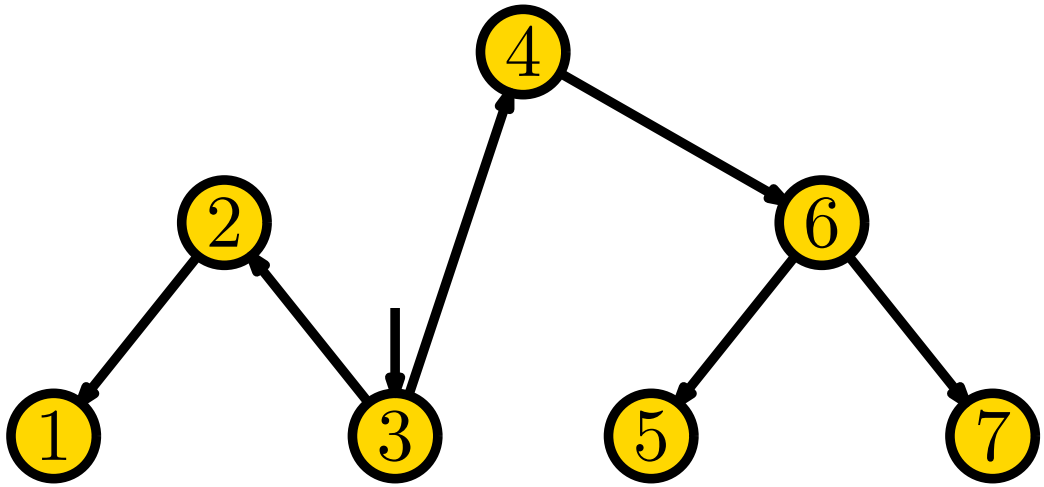
- Keys
- Pointer Structure
- Key Access
- Adjustments



Dynamic Dictionaries

The Setting:

- Keys
- Pointer Structure
- Key Access
- Adjustments
- Online



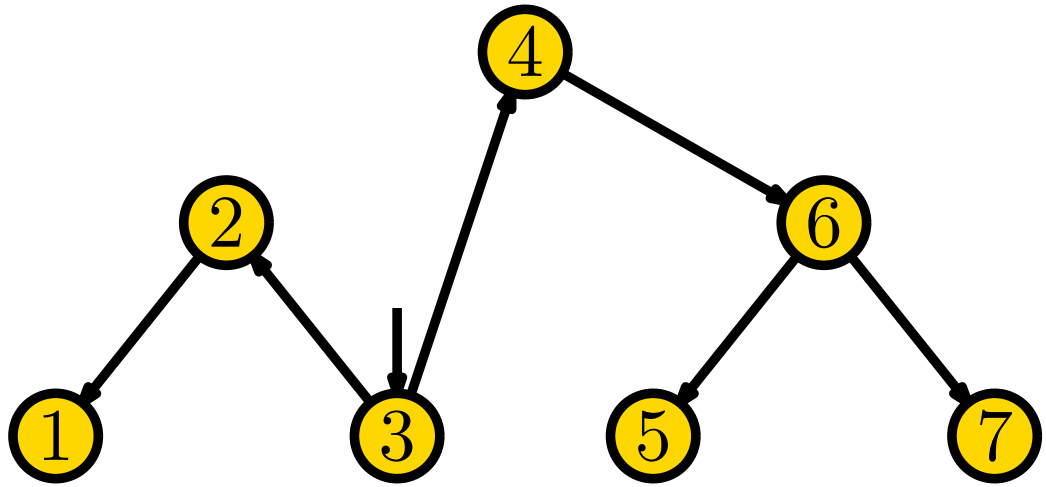
Dynamic Dictionaries

The Setting:

- Keys
- Pointer Structure
- Key Access
- Adjustments
- Online

The Goal:

- Dynamic Optimalty



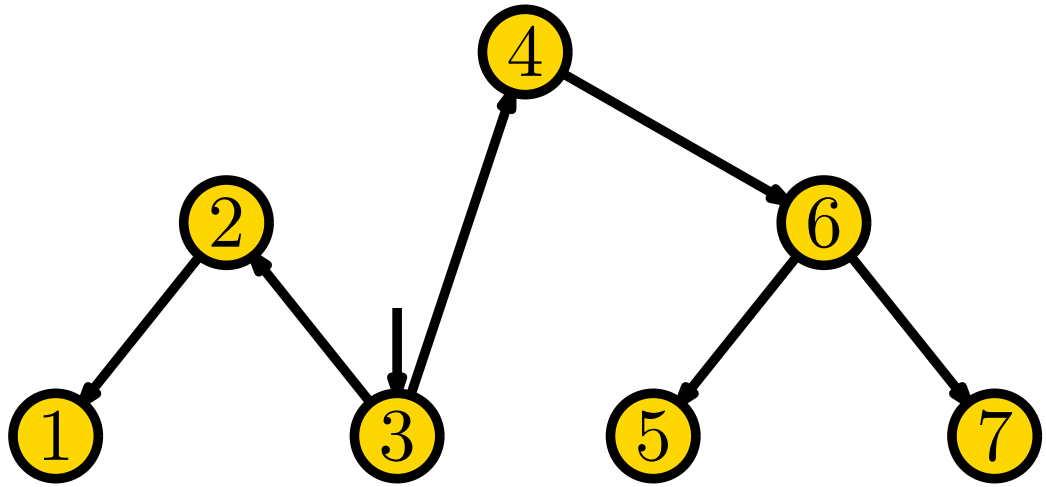
Dynamic Dictionaries

The Setting:

- Keys
- Pointer Structure
- Key Access
- Adjustments
- Online

The Goal:

- Dynamic Optimalty
- Minimise: $\frac{\text{Best Online Cost}}{\text{Best Offline Cost}}$



Dynamic Dictionaries

The Setting:

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- Pointer Structure
- Key Access
- Adjustments
- Online

The Goal:

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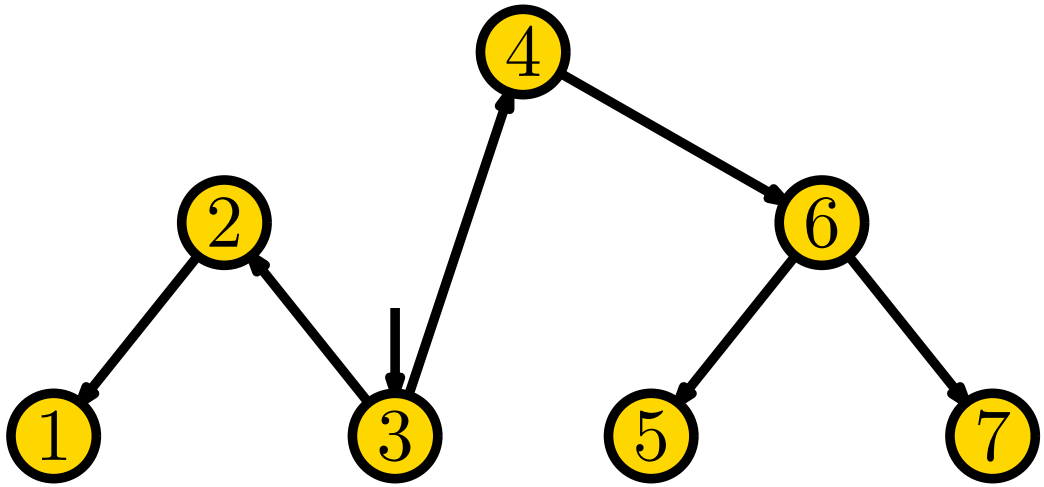
- Minimise: $\frac{\text{Best Online Cost}}{\text{Best Offline Cost}}$

Towards Networks:

- Pairwise Access



Dynamic Dictionaries

The Setting:

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The Goal:

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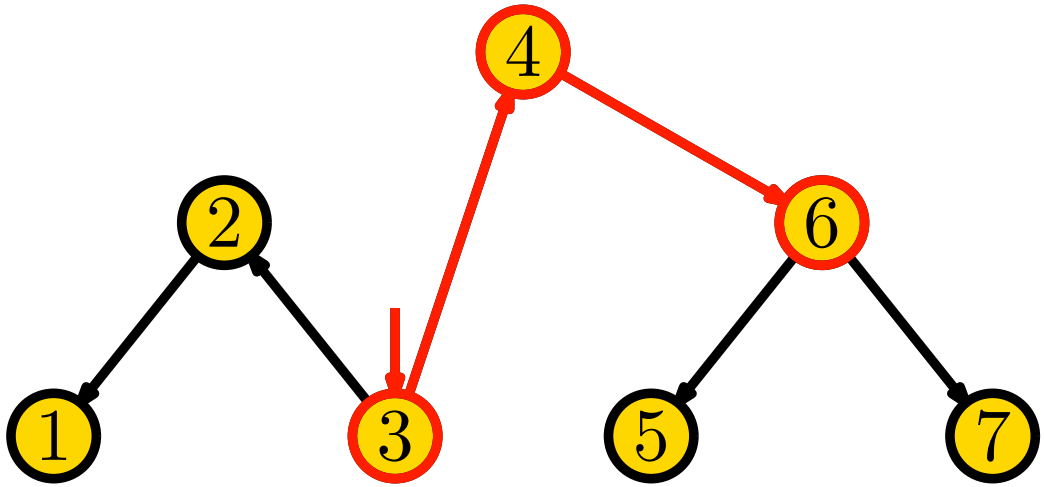
- Minimise: $\frac{\text{Best Online Cost}}{\text{Best Offline Cost}}$

Towards Networks:

- Pairwise Access

Access 6



Dynamic Dictionaries

The Setting:

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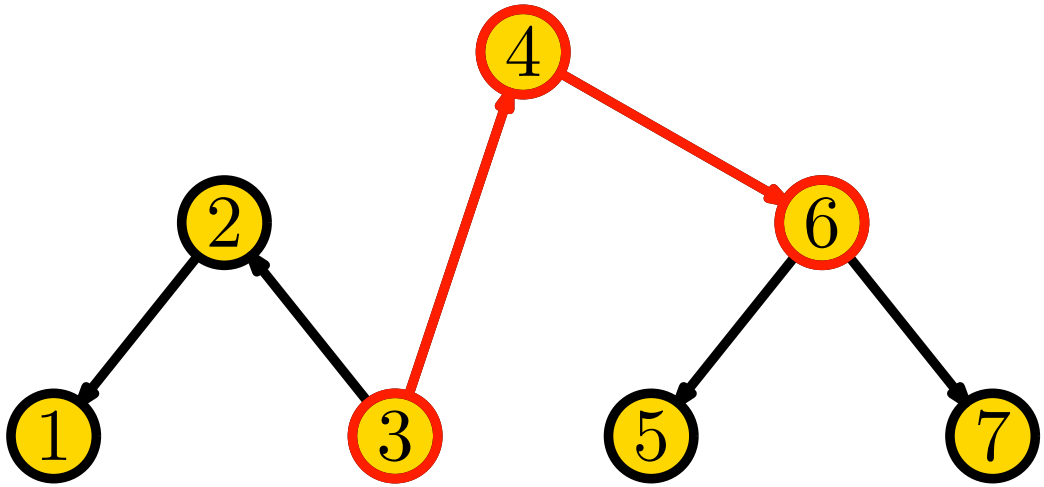
Towards Networks:

- Pairwise Access

~~Access 6~~

Communicate $(3, 6)$



Dynamic Dictionaries

The Setting:

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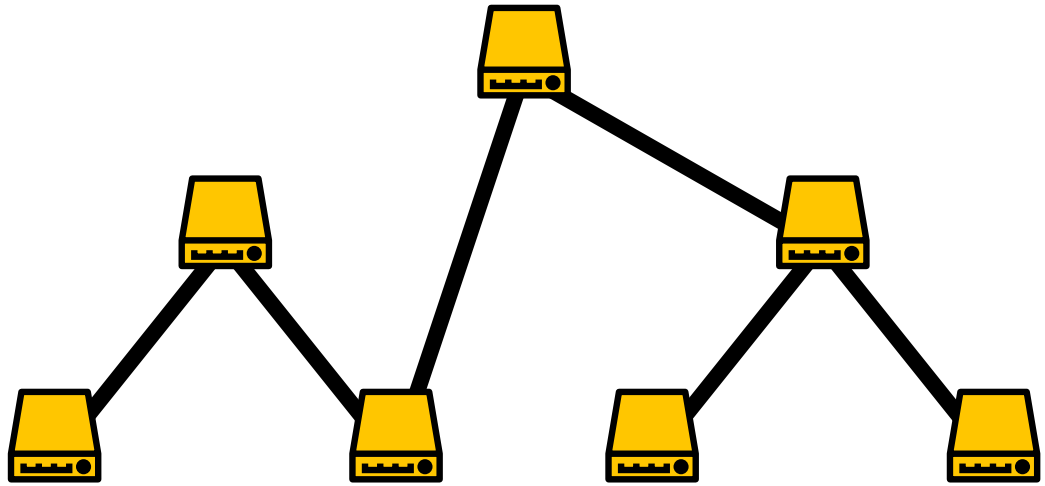
- Adjustments
- Online

The Goal:

- Dynamic Optimalty
- Minimise: $\frac{\text{Best Online Cost}}{\text{Best Offline Cost}}$

Towards Networks:

- Pairwise Access
- Ignore Searching/Routing

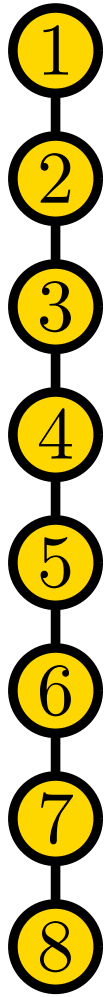


A Simple$^{?}$ Network

A Simple[?] Network

The Dictionary:

- Linked List



A Simple[?] Network

The Dictionary:

- Linked List
- Access in Front

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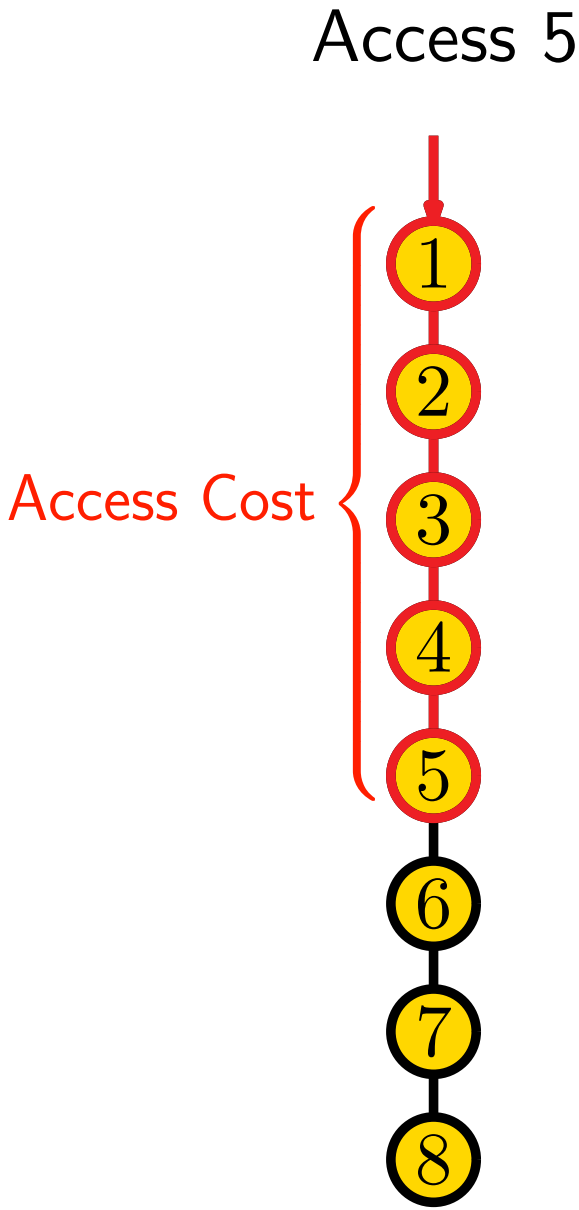


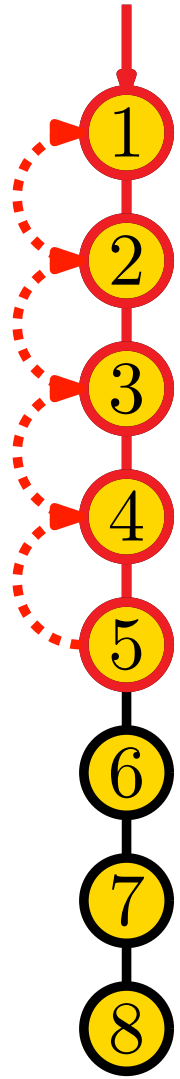
A Simple[?] Network

The Dictionary:

- Linked List
- Access in Front
- Move-to-Front Algorithm

Access 5

Move Cost



A Simple[?] Network

The Dictionary:

- Linked List
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Access 5

A Simple$^{?}$ Network

The Dictionary:

- Linked List
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  $\rightarrow$ Dynamically Optimal!

A Simple[?] Network

The Dictionary:

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- Access in Front
- Move-to-Front Algorithm
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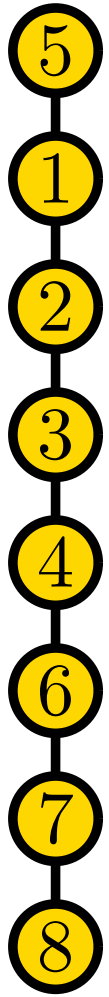
The Network:

- Pairwise Access



A Simple[?] Network

The Dictionary:

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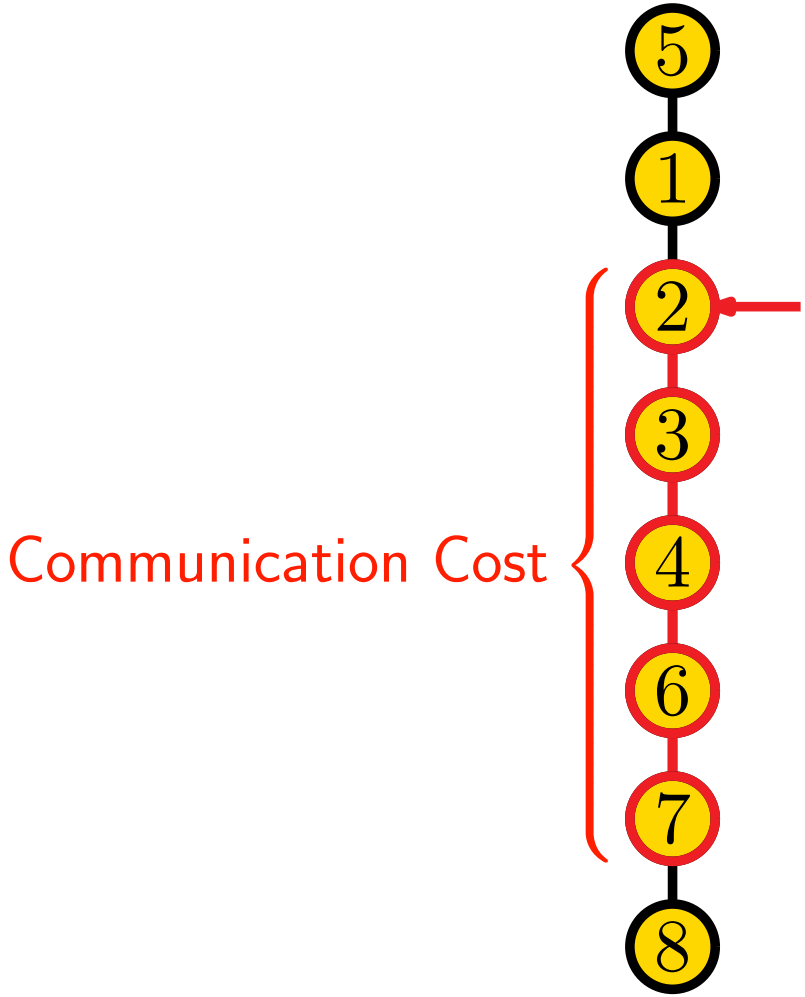
The Network:

- Pairwise Access

Communicate (2,7)



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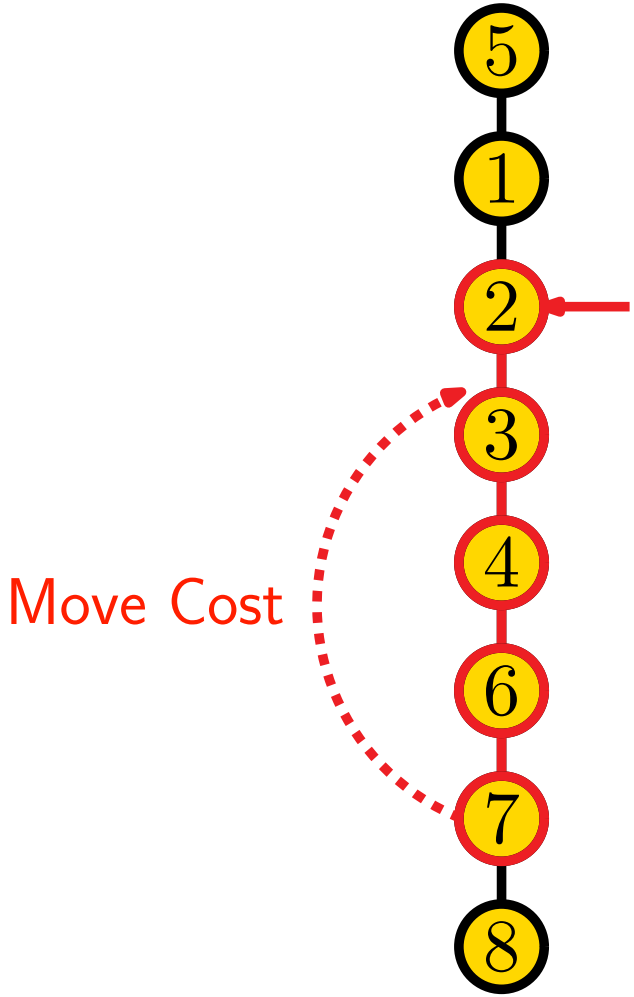

A Simple[?] Network

The Dictionary:

- Linked List
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  → Dynamically Optimal!

The Network:

- Pairwise Access
- Move-to-Where?

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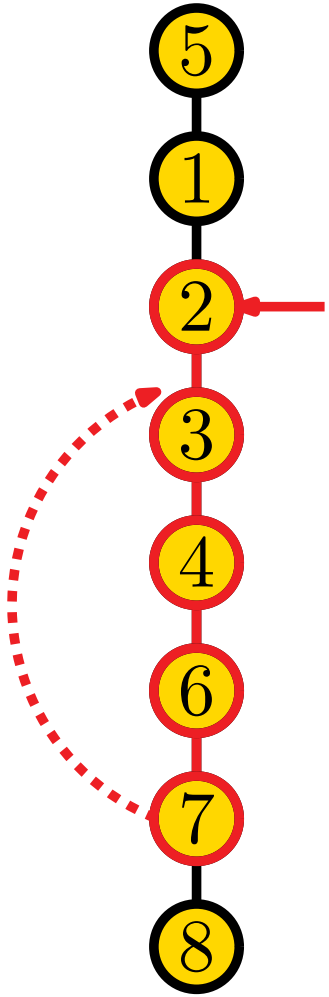

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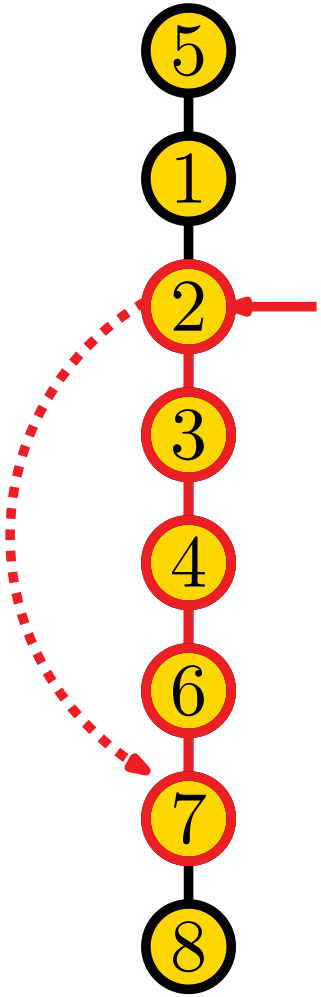

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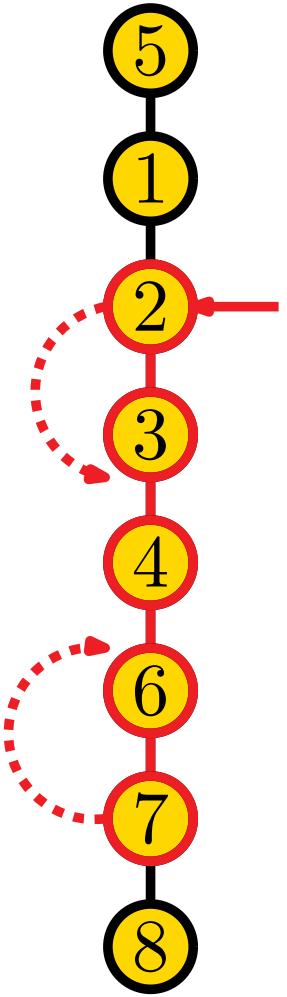

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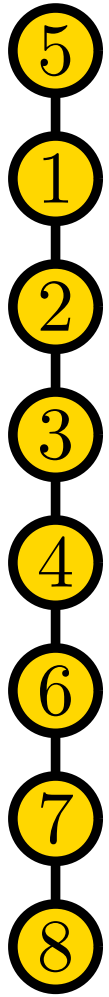

A Simple[?] Network

The Dictionary:

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The Network:

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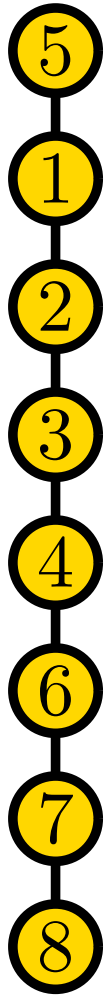
Not So Simple:

- Co-locating can Backfire



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  → Dynamically Optimal!

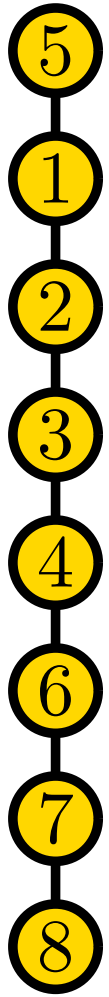
The Network:

- Pairwise Access
- Move-to-Where?

Not So Simple:

- Co-locating can Backfire

Communication:



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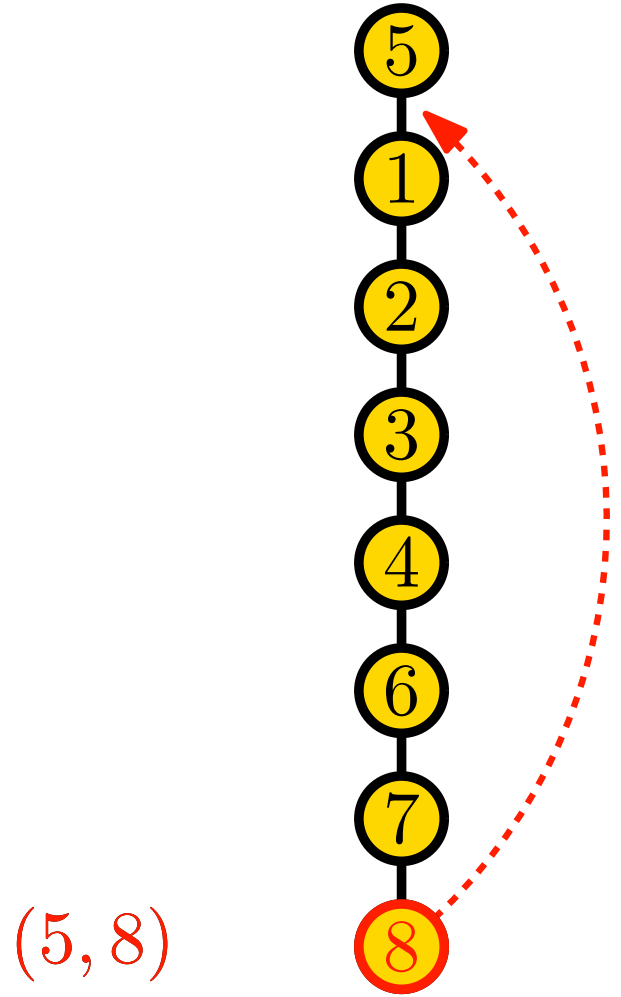

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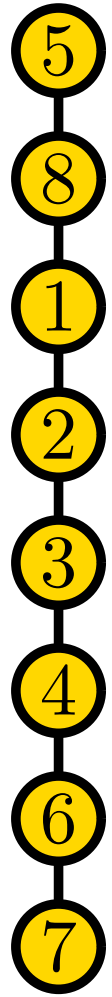

$(5, 8)$

A Simple[?] Network

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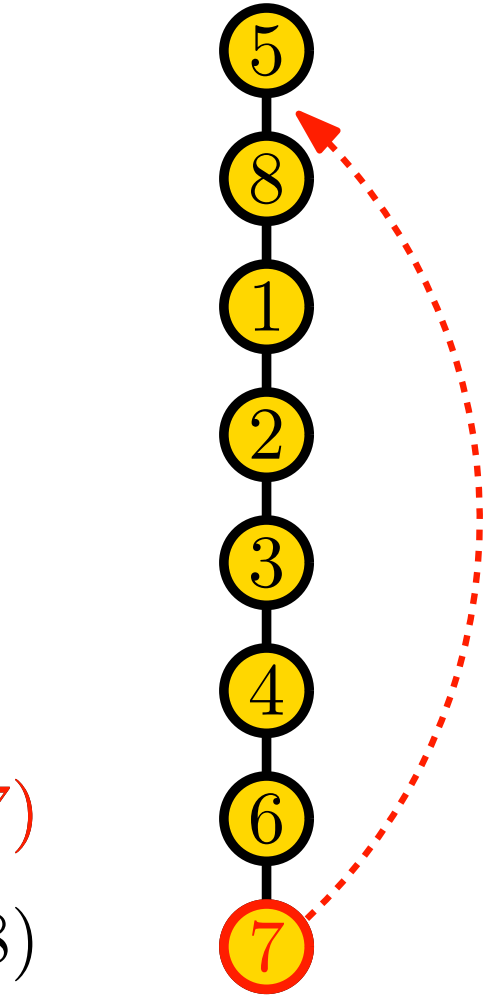

$(5, 7)$

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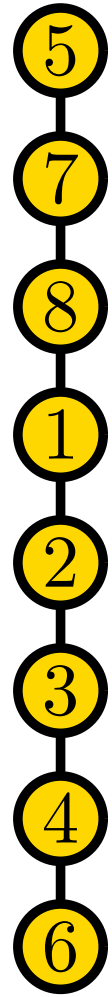

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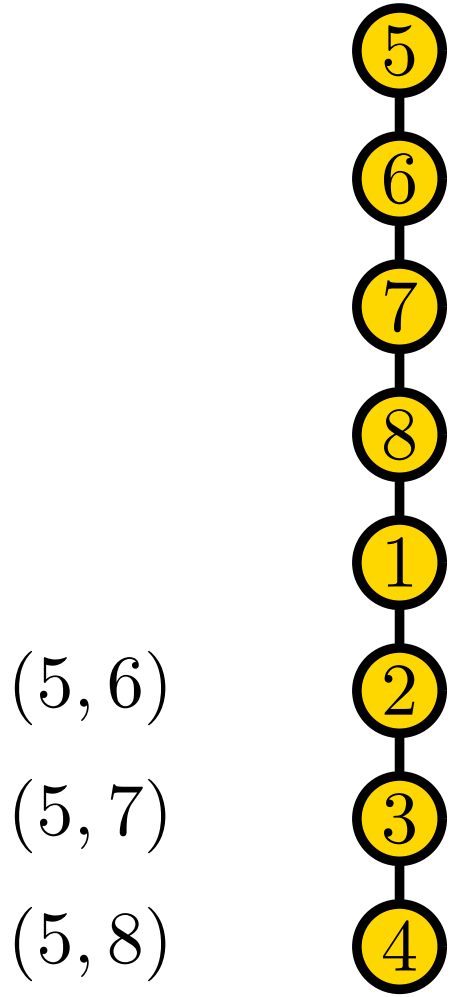

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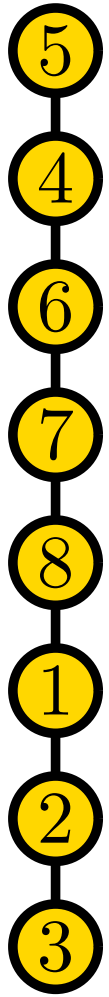
Communication:

$(5, 4)$

$(5, 6)$

$(5, 7)$

$(5, 8)$



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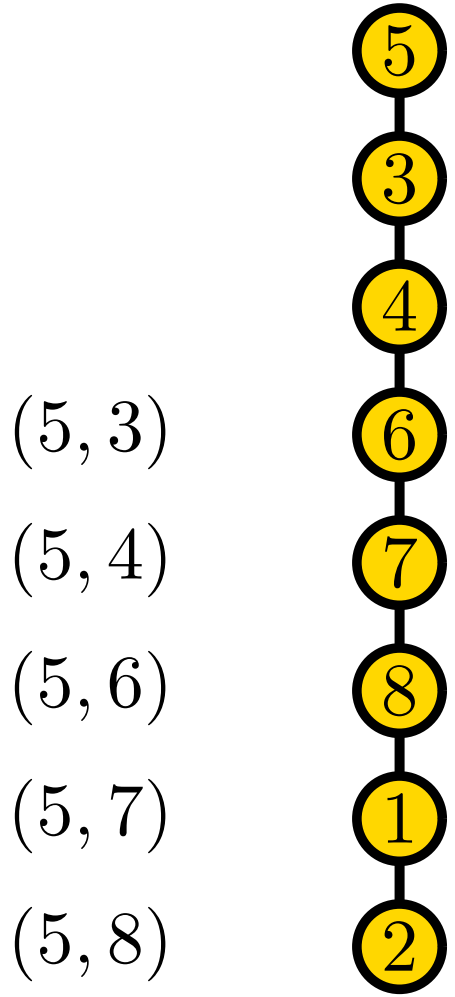

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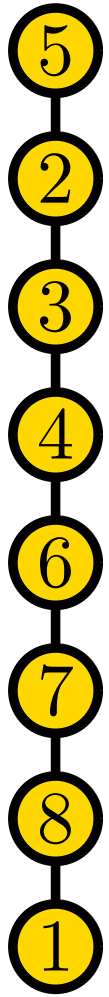
Communication:

$(5, 2)$
$(5, 3)$
$(5, 4)$
$(5, 6)$
$(5, 7)$
$(5, 8)$



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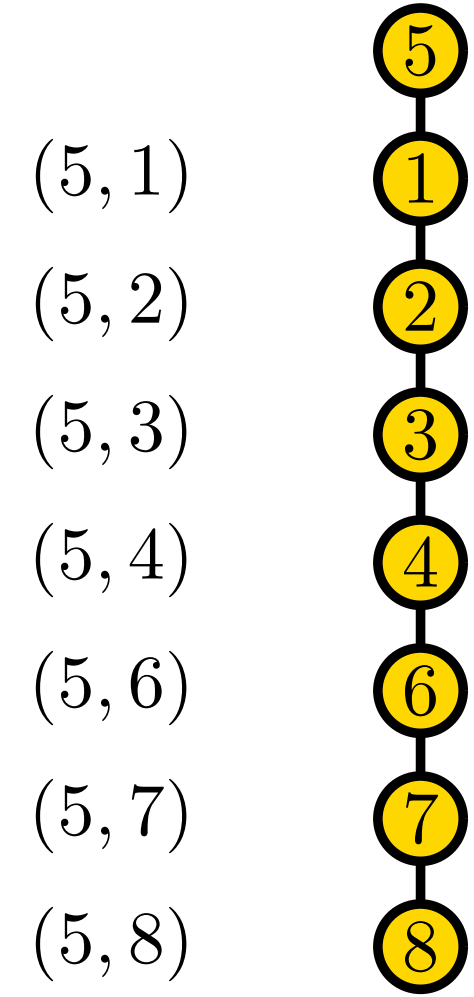

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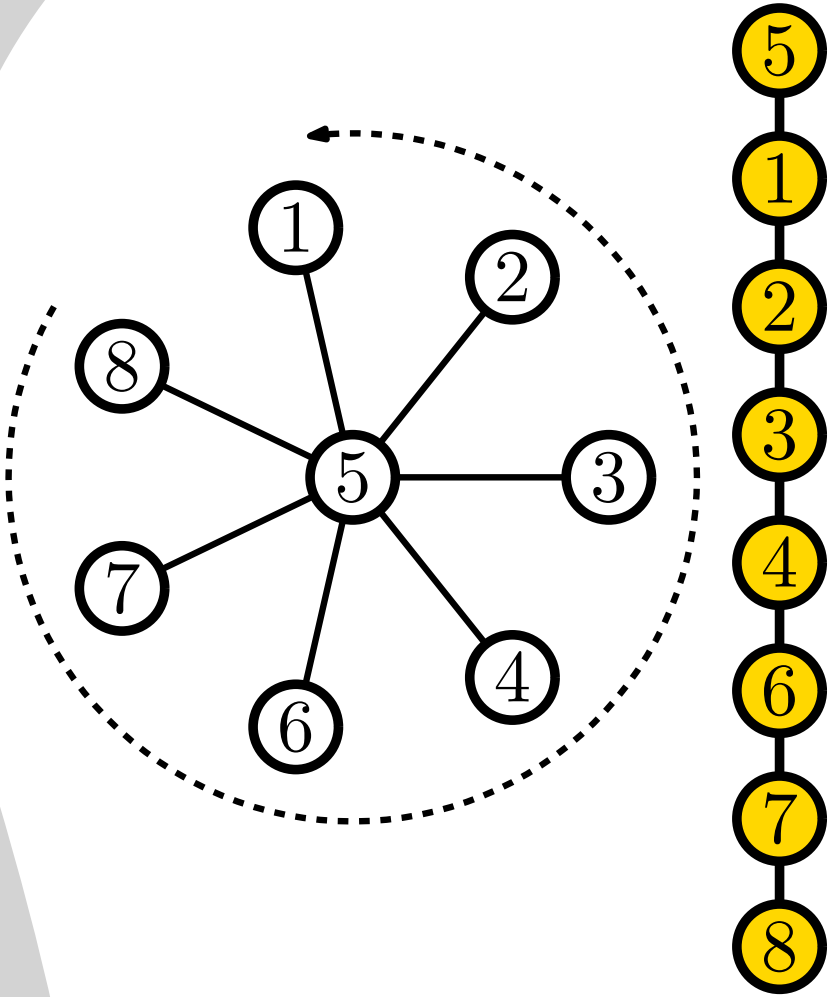

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- Same Conclusion by Olver et al.

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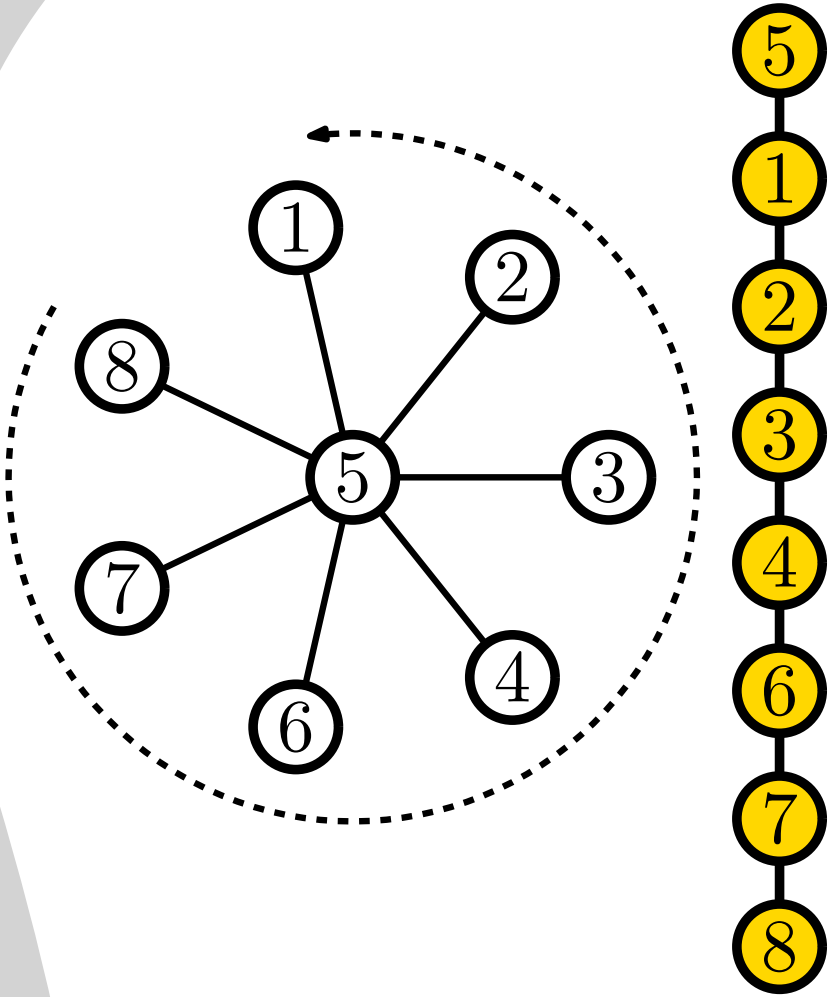
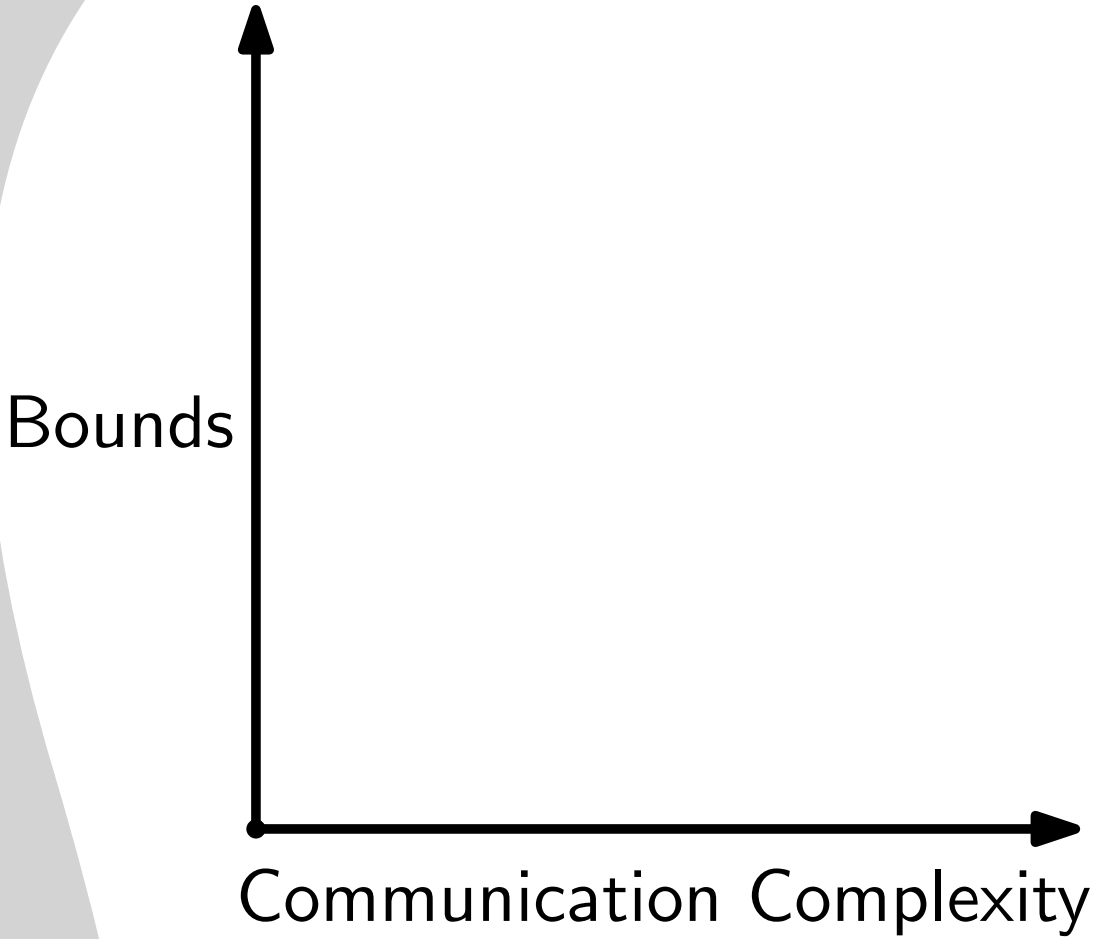


Results

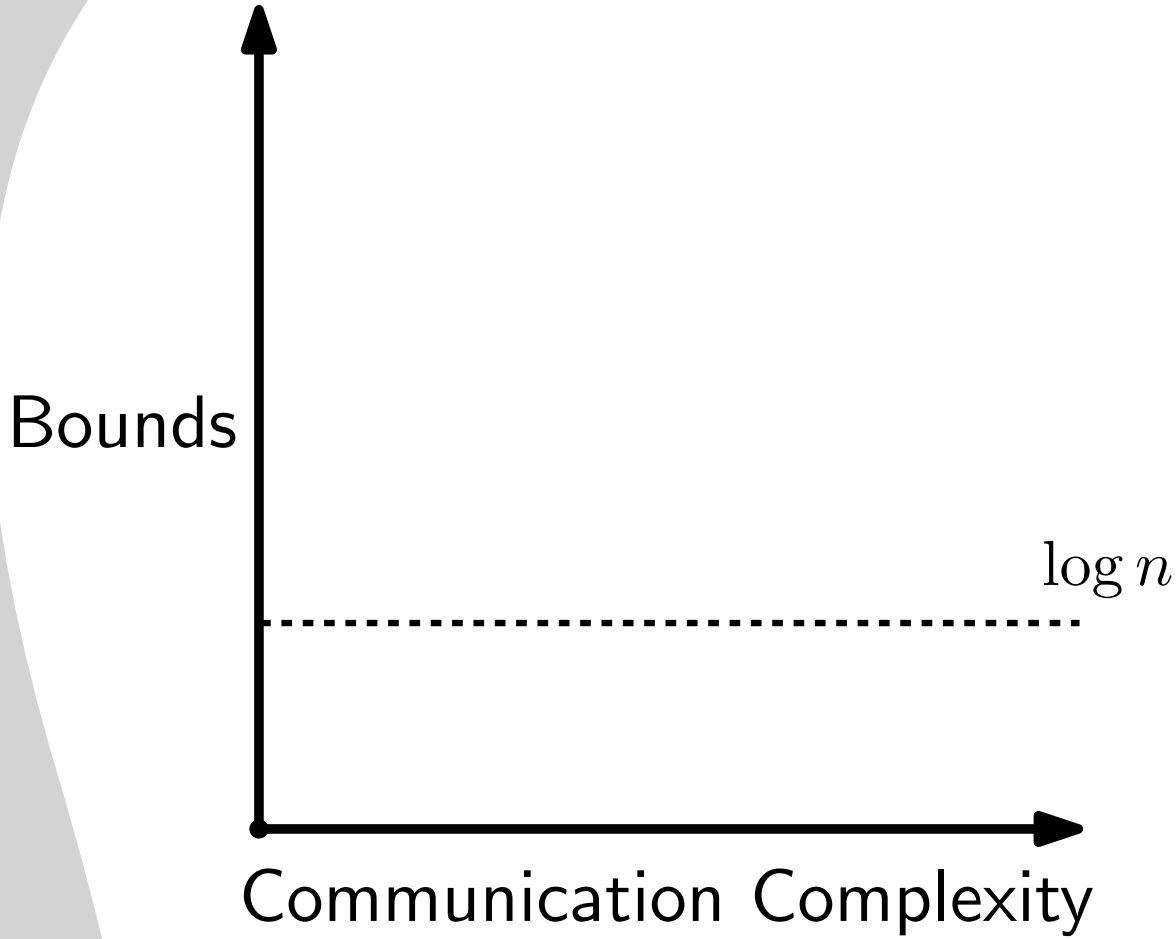
Results



Results

Grid Networks:

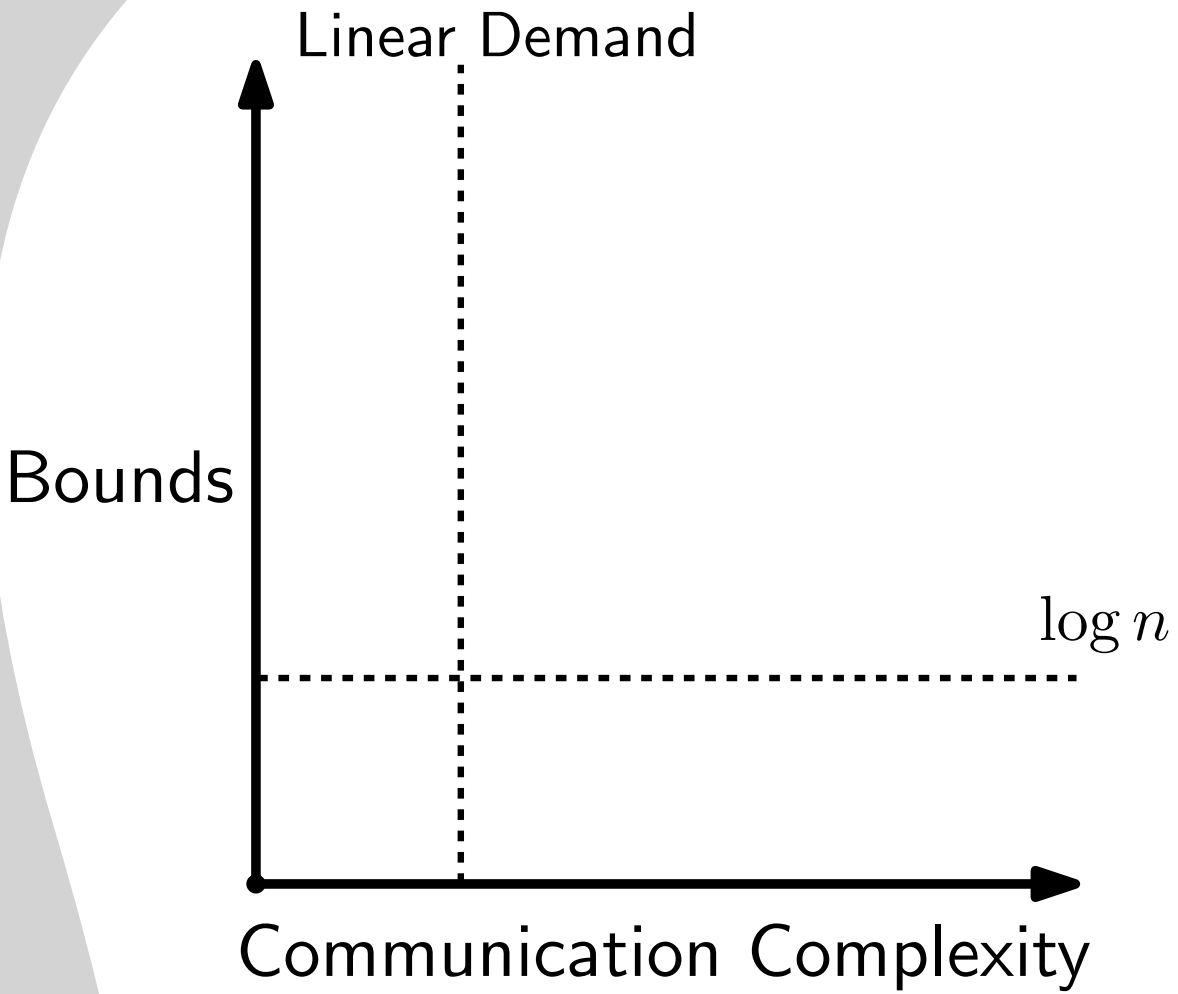
- Bounds on Competitive Ratio



Results

Grid Networks:

- Bounds on Competitive Ratio
- Restricted Communication



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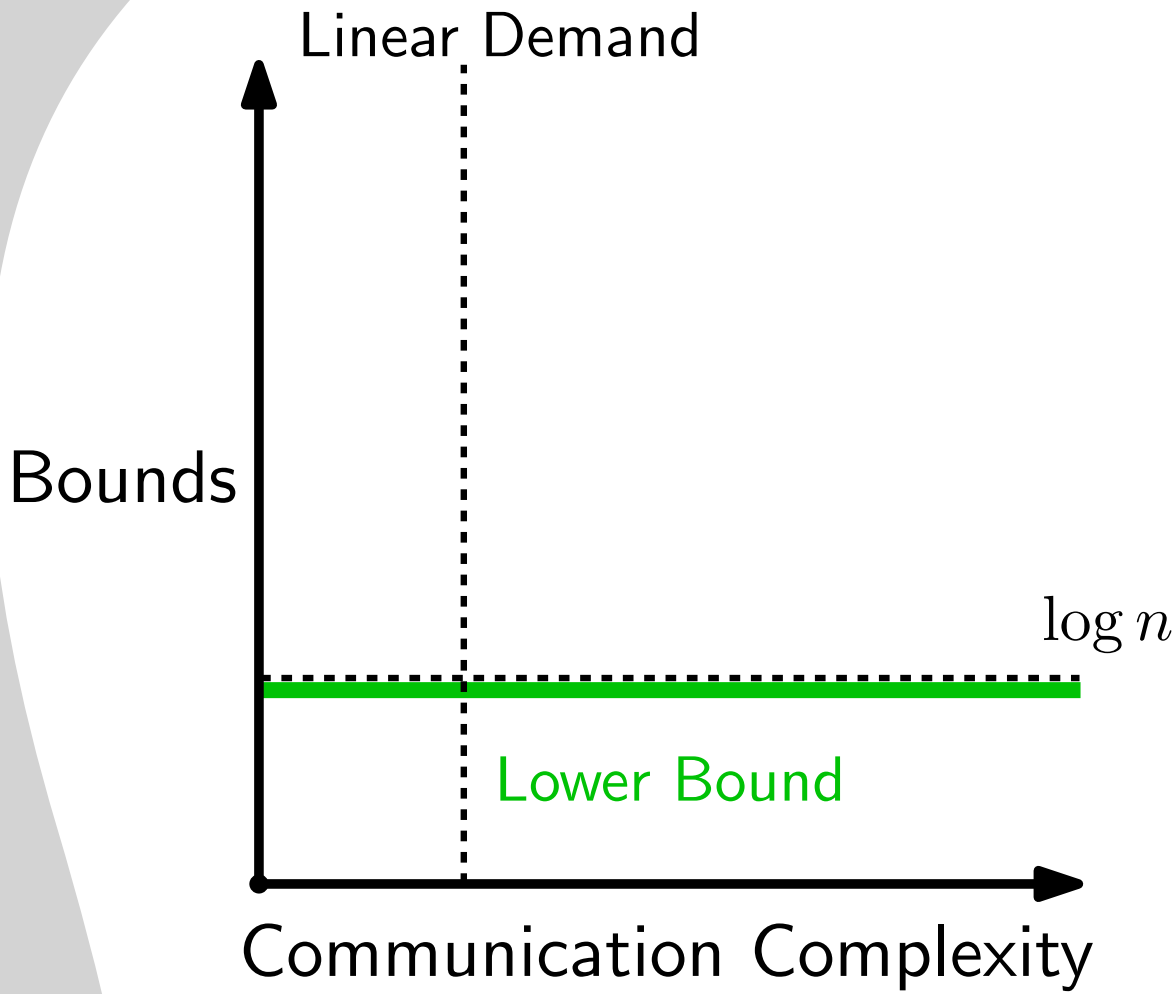

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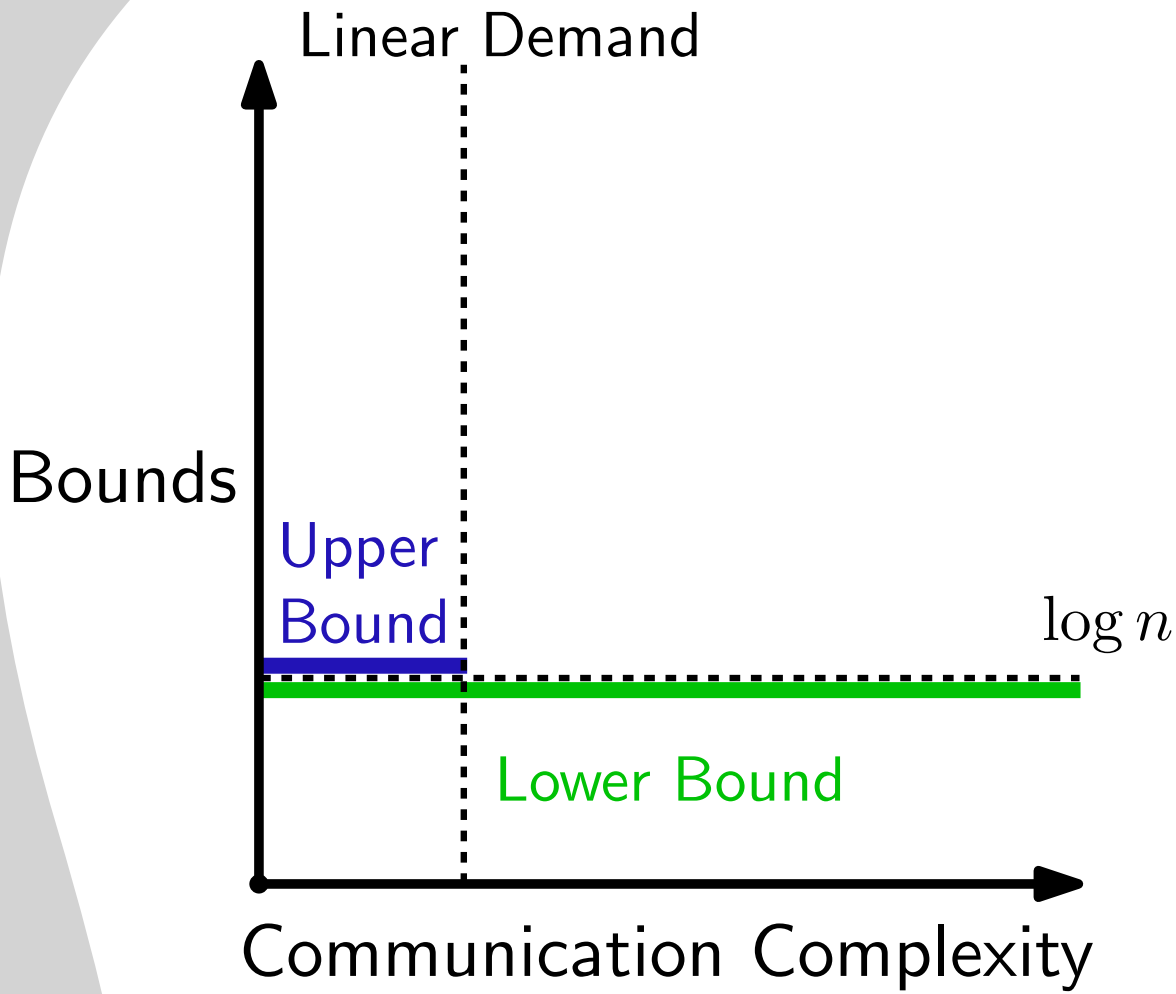

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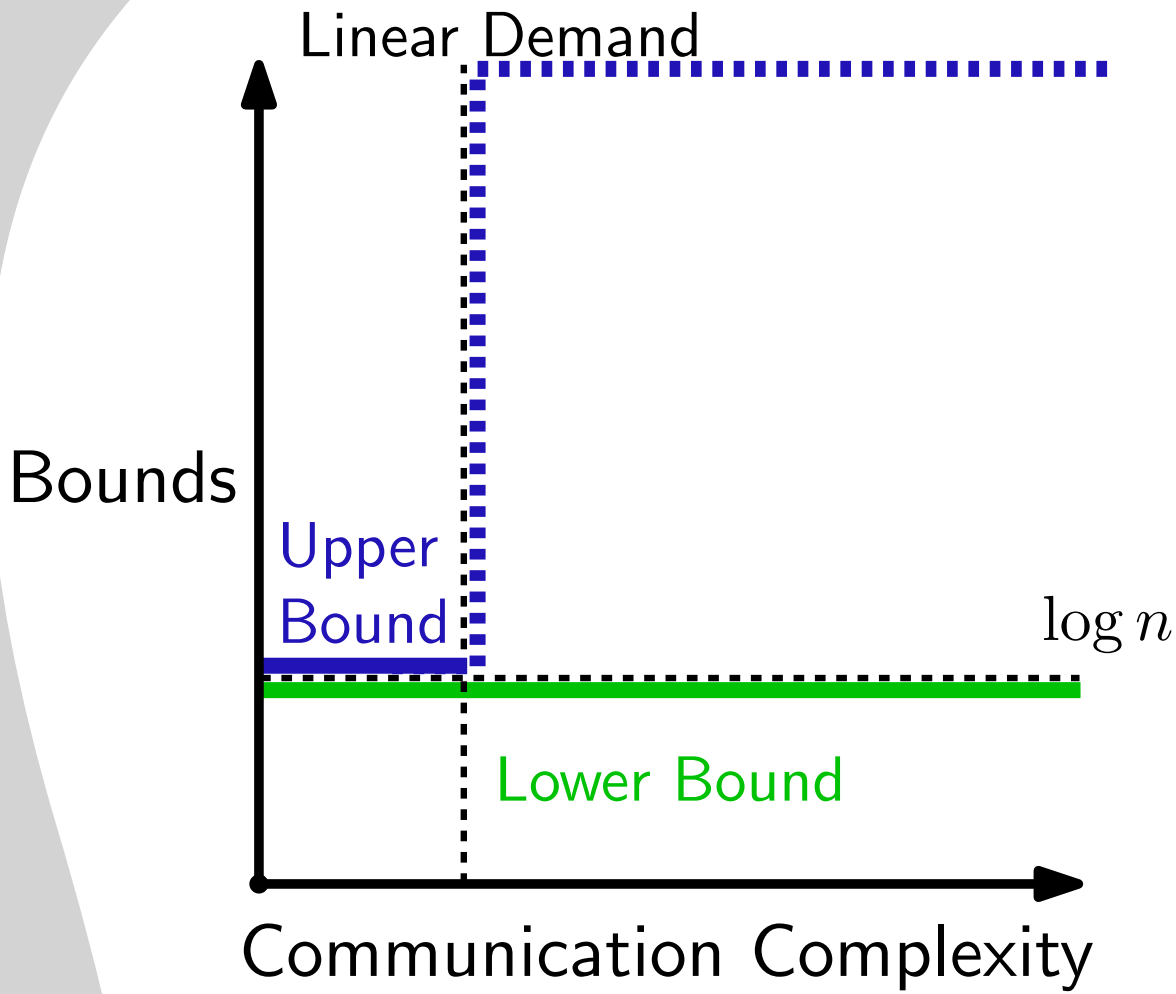

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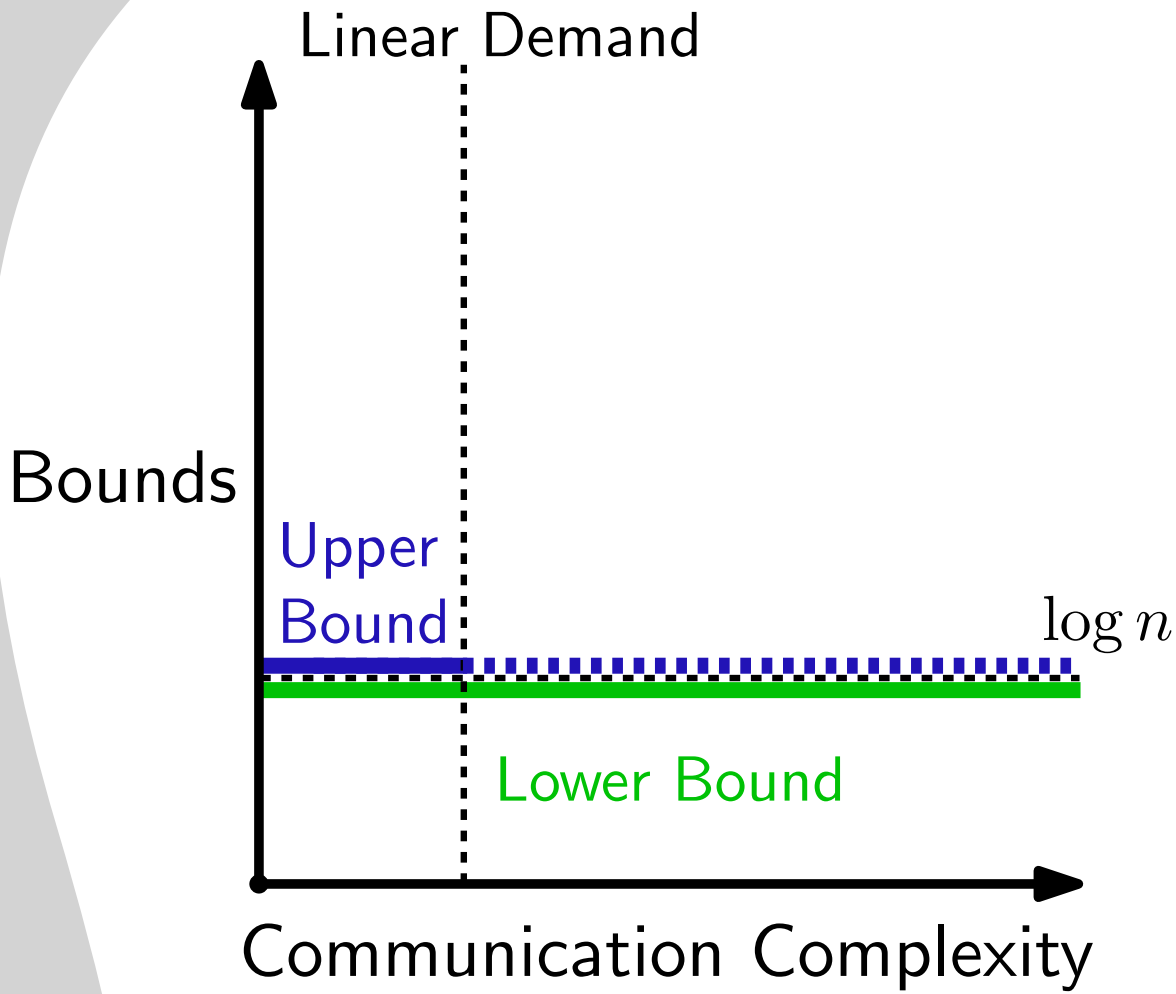

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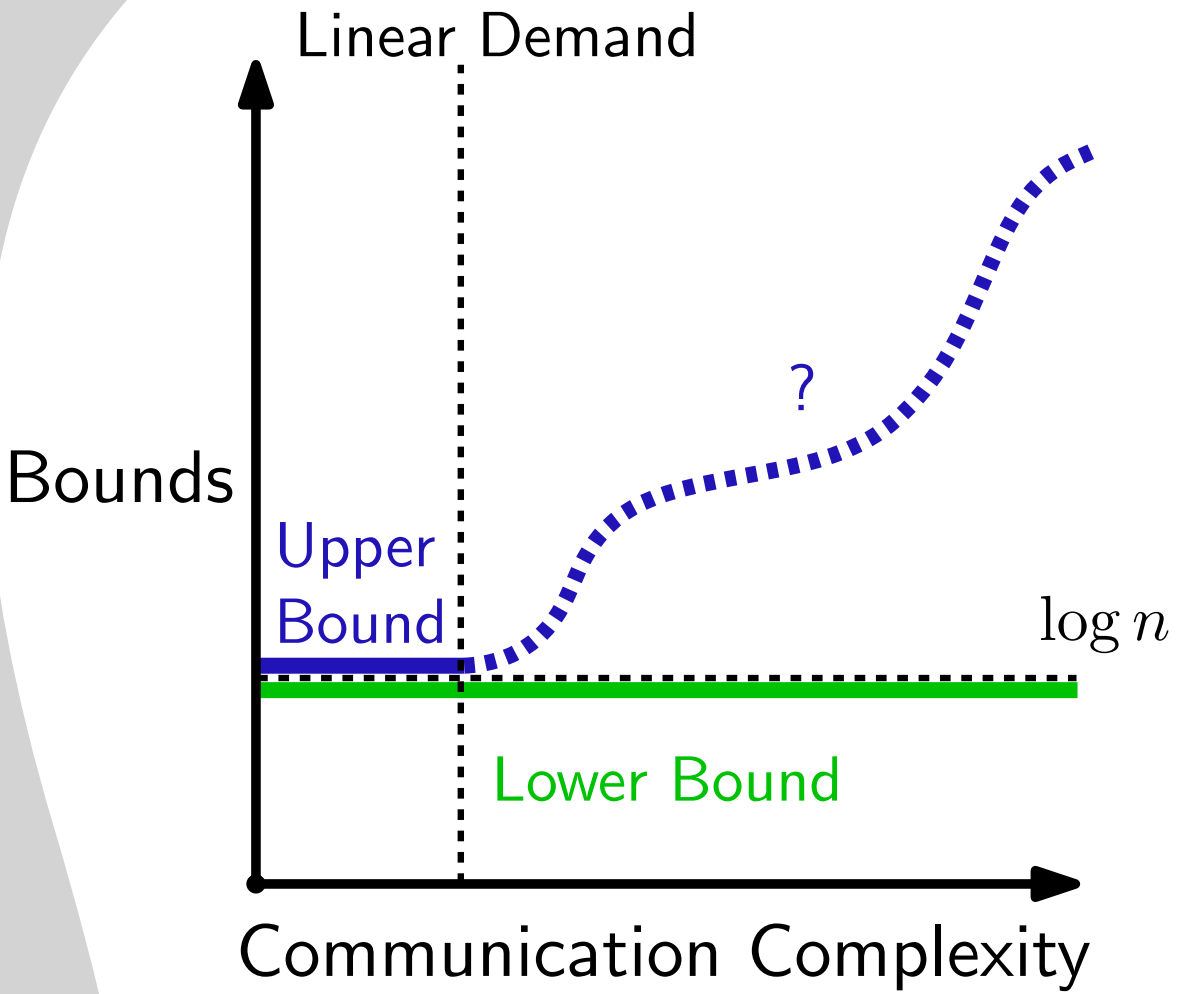


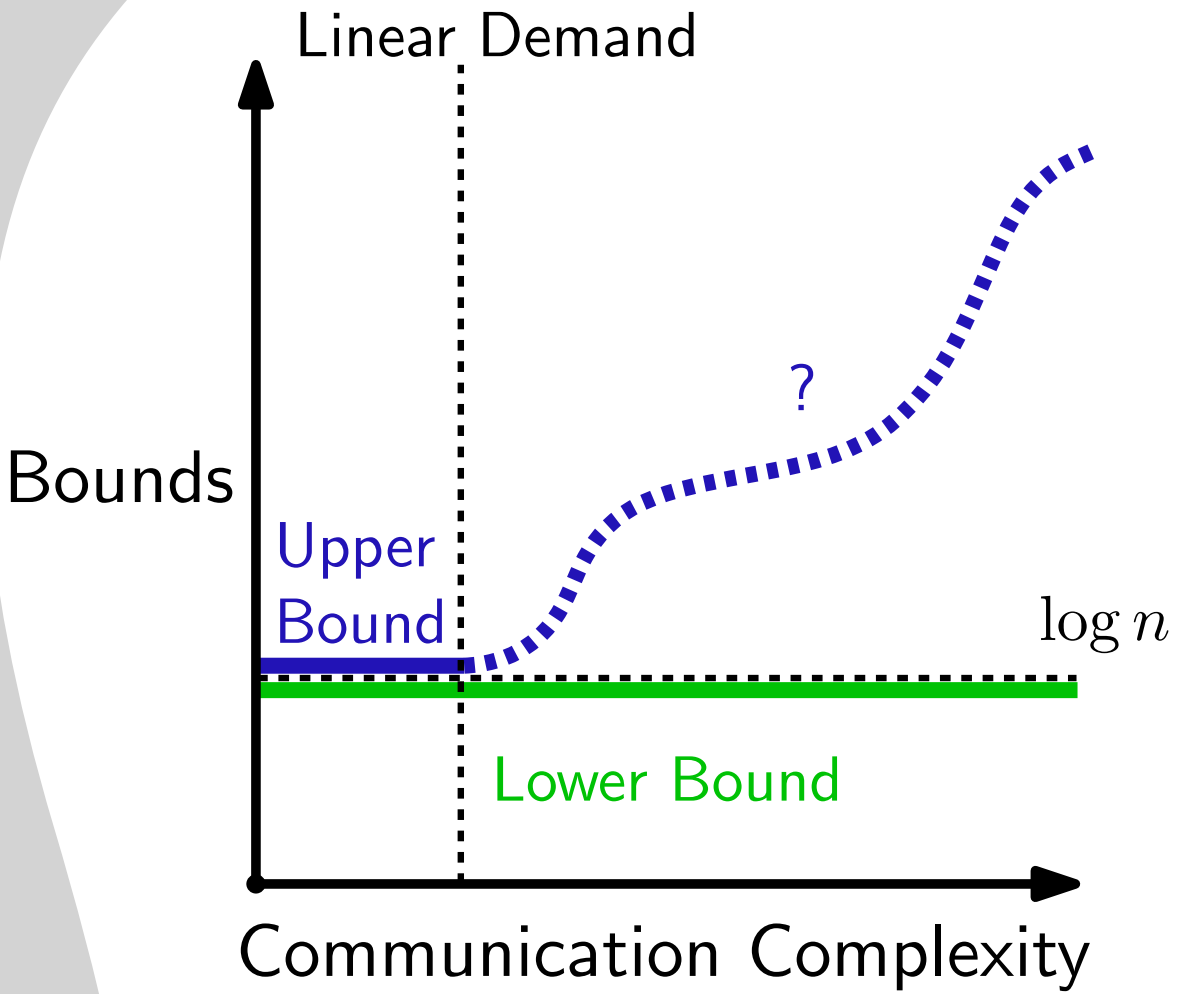
Results

Grid Networks:

- Bounds on Competitive Ratio
- Restricted Communication

Line Networks:

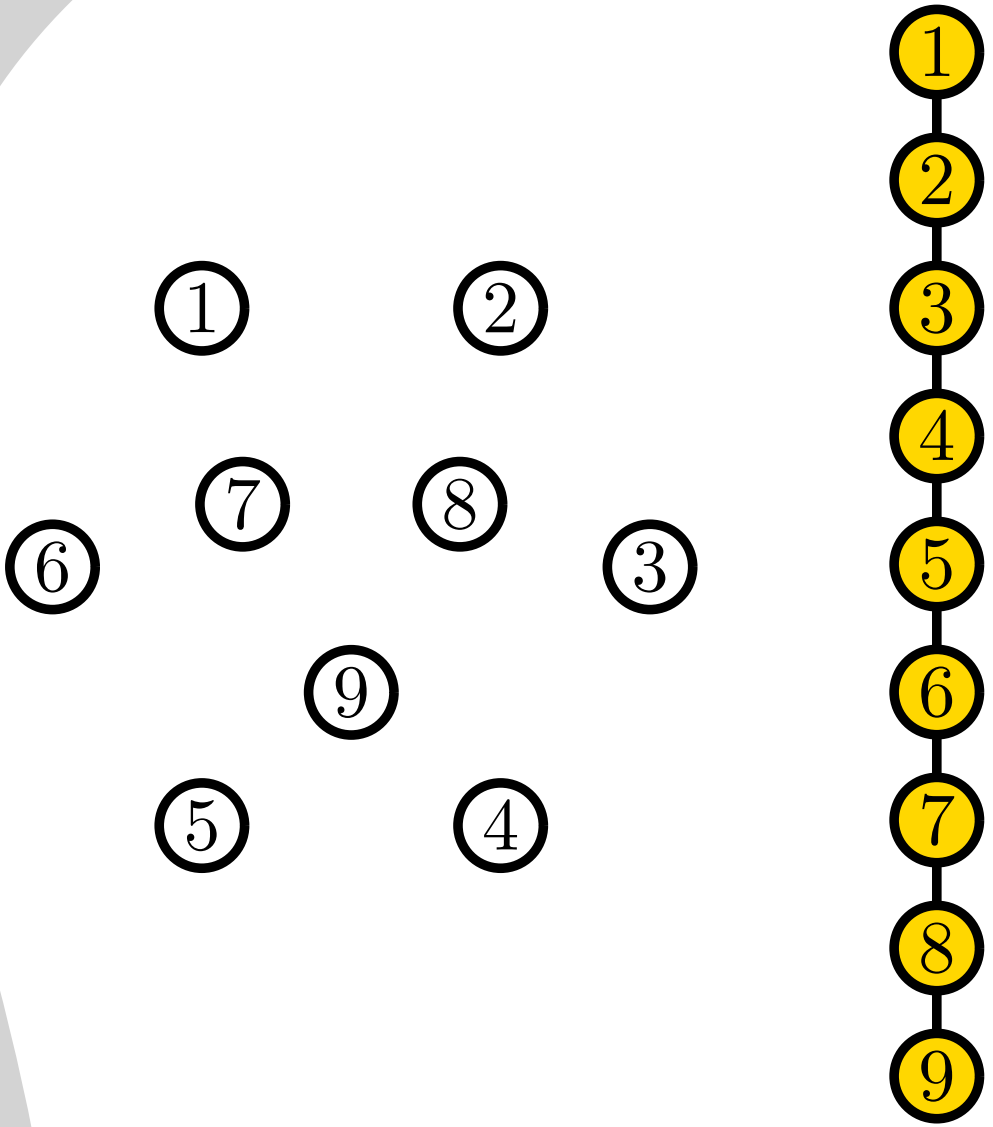
- Distributed Implementation



Lower Bound

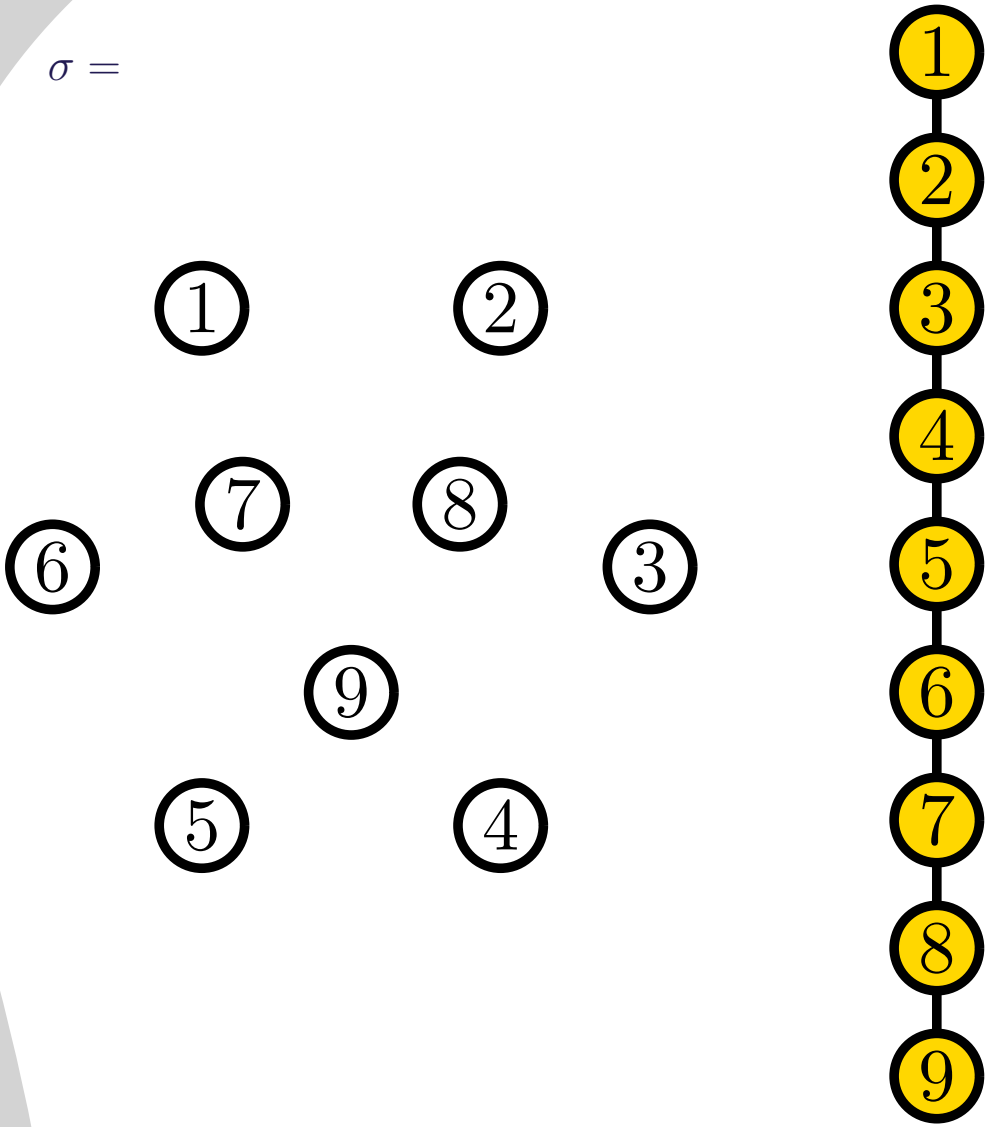
Lower Bound

Lower Bound



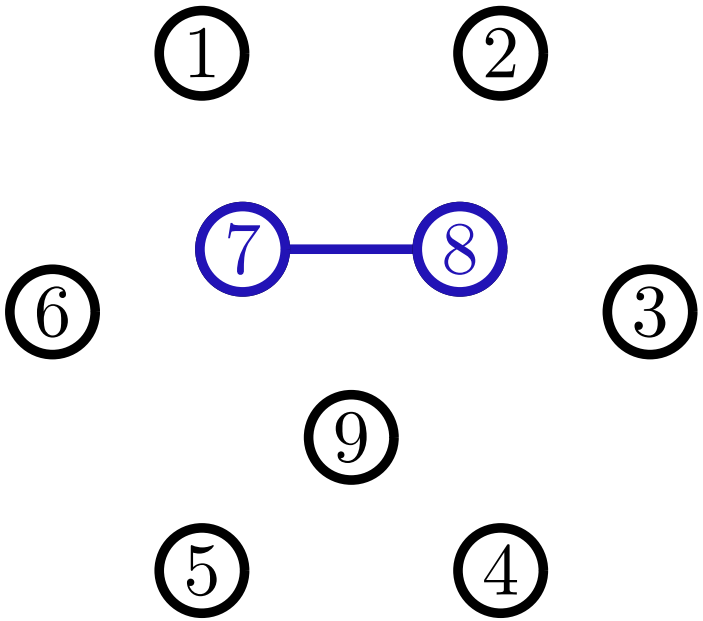
Lower Bound

$\sigma =$



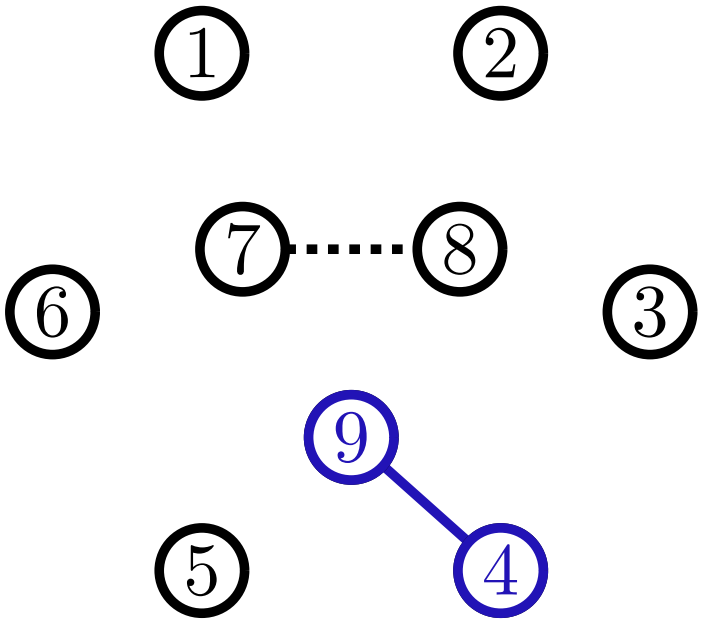
Lower Bound

$\sigma = (7, 8)$



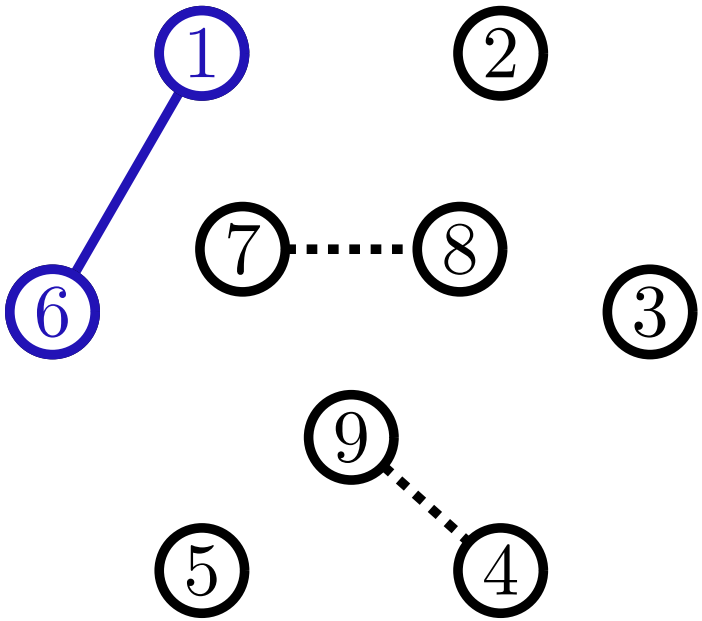
Lower Bound

$\sigma = (7,8)(4,9)$



Lower Bound

$\sigma = (7,8)(4,9)(1,6)$



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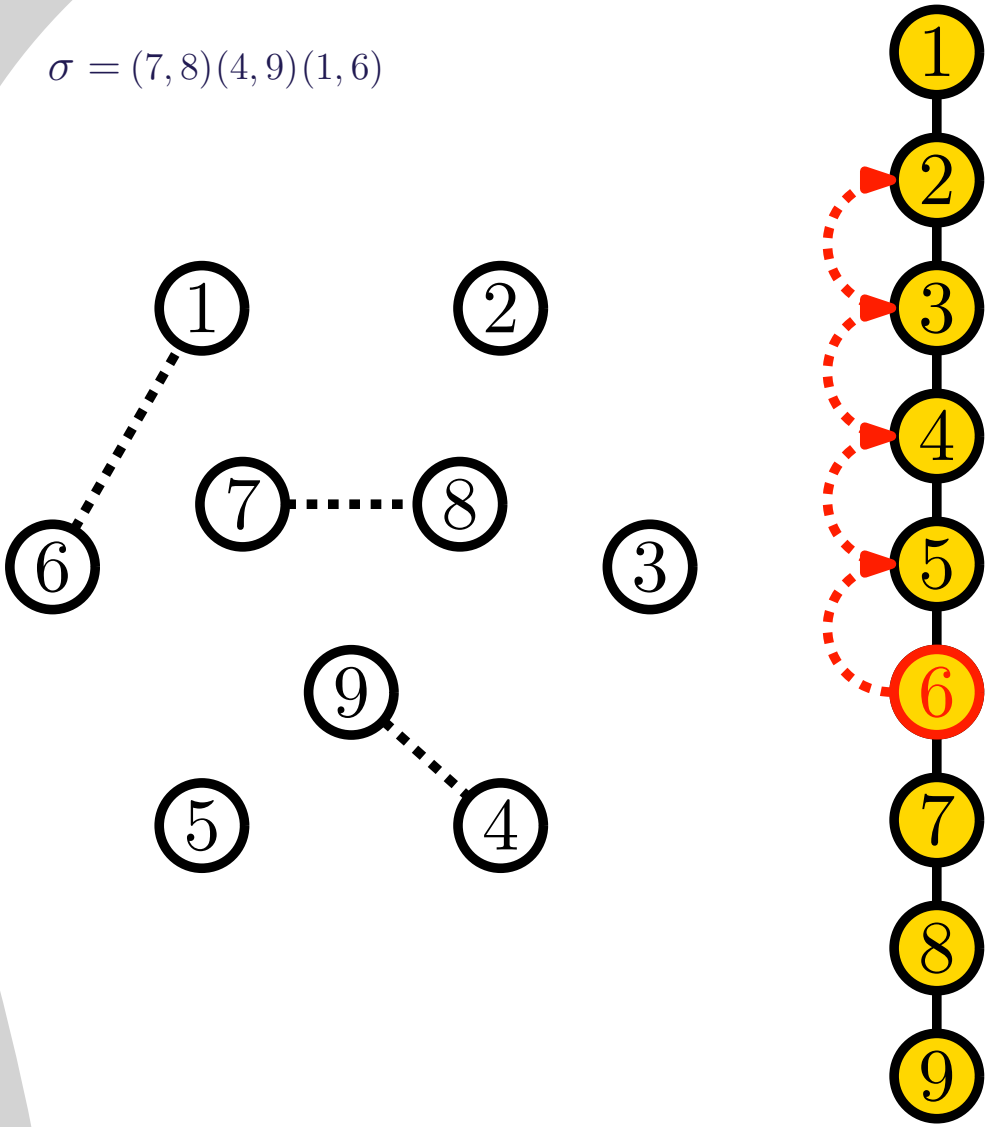

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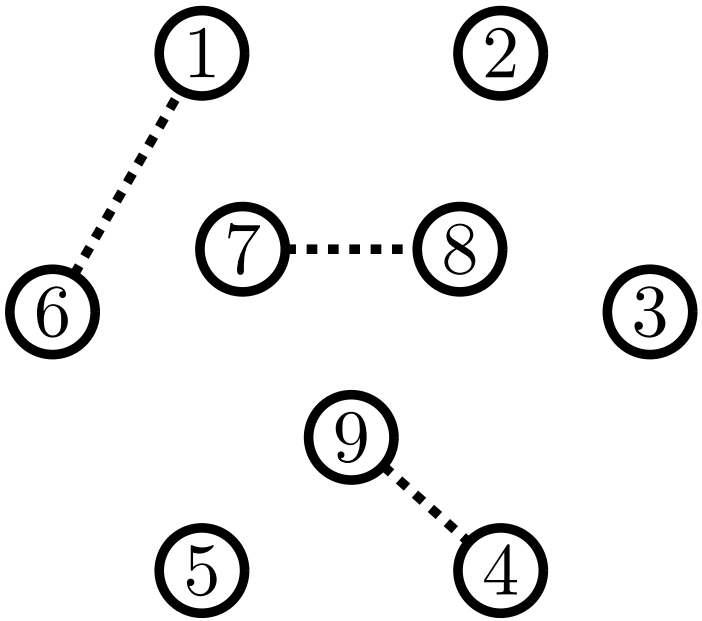

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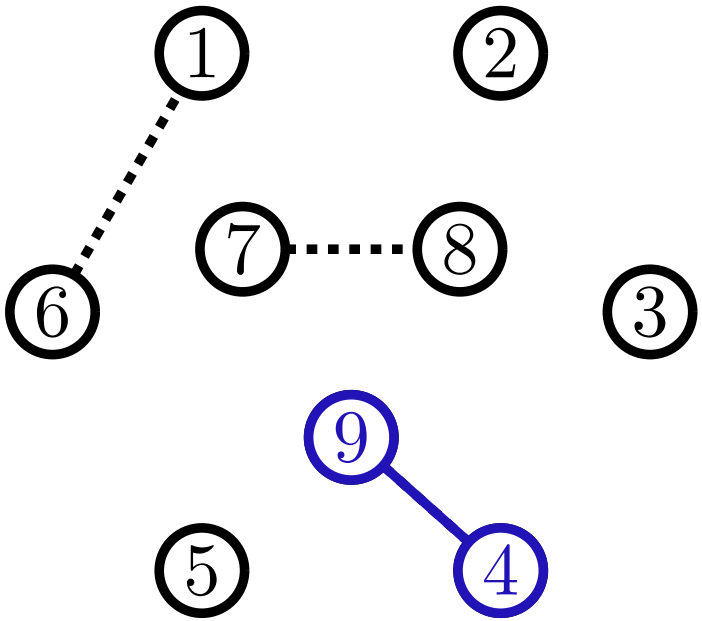

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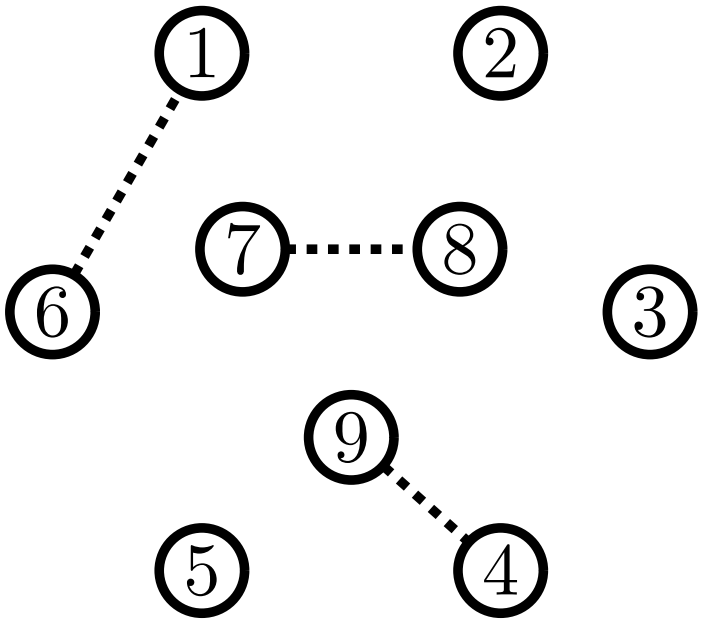

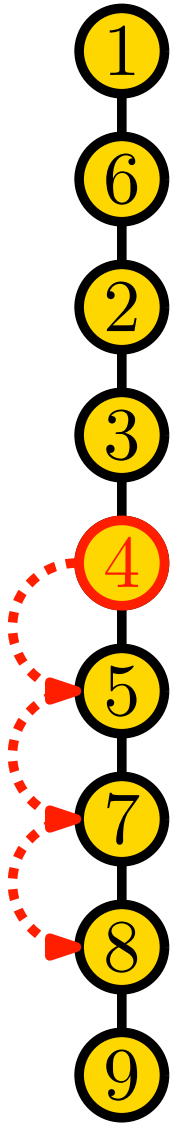

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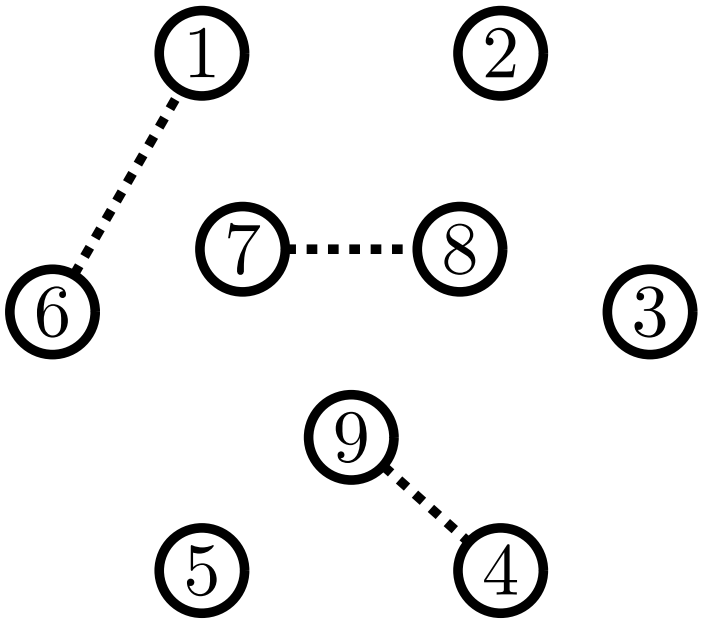

Lower Bound

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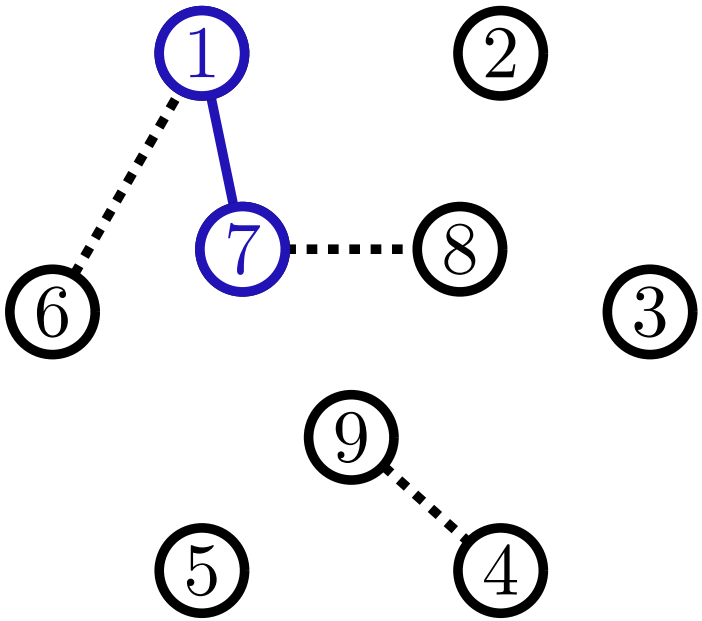

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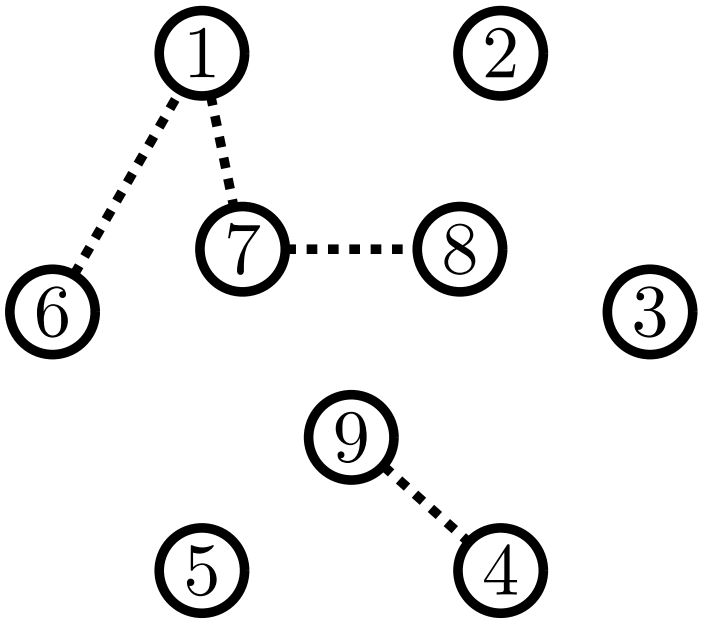

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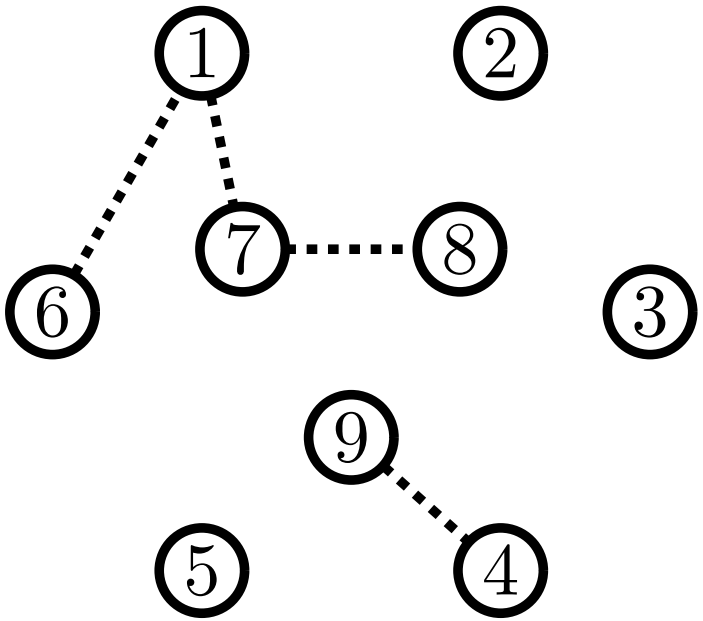

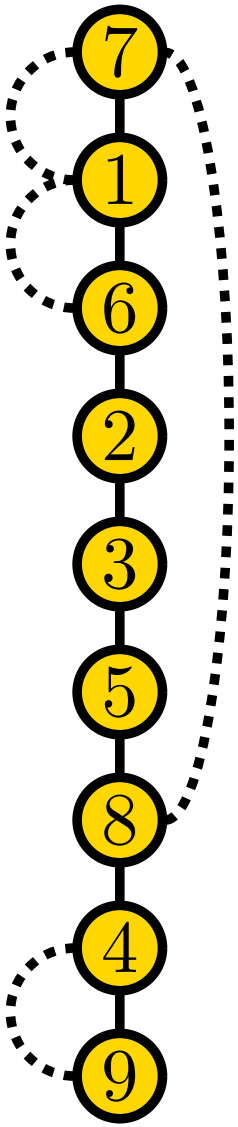

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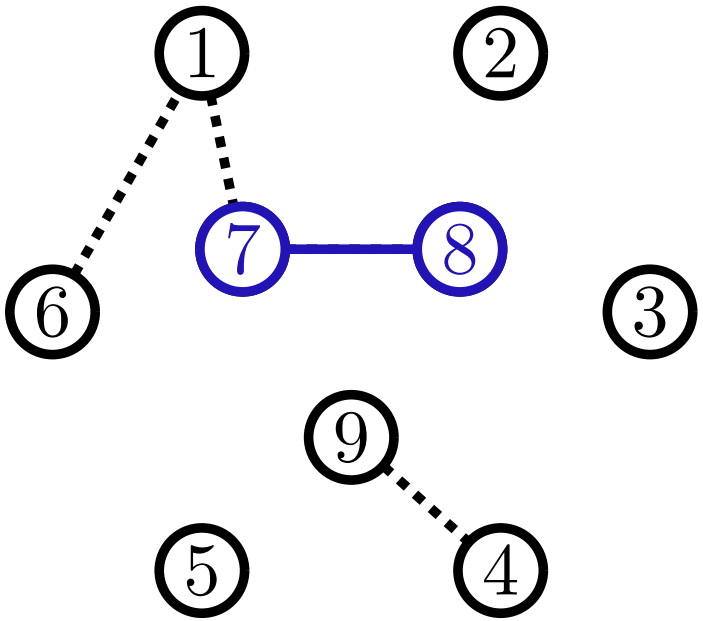

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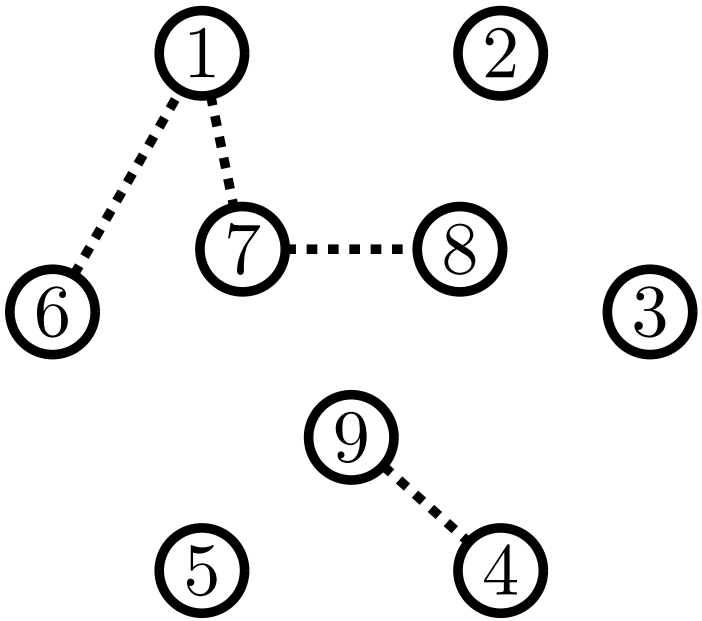

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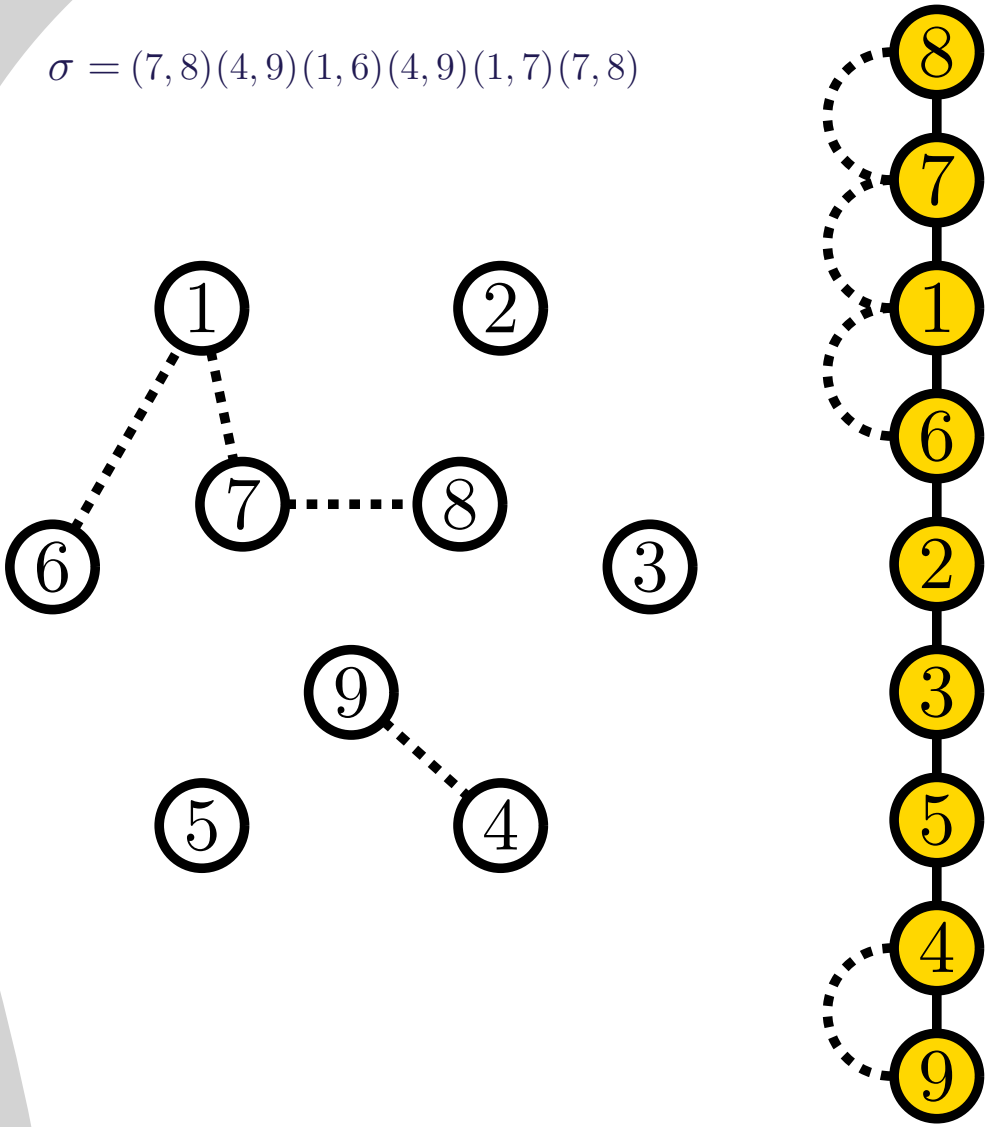

Lower Bound

The Strategy:

- Exploit Bad Edges

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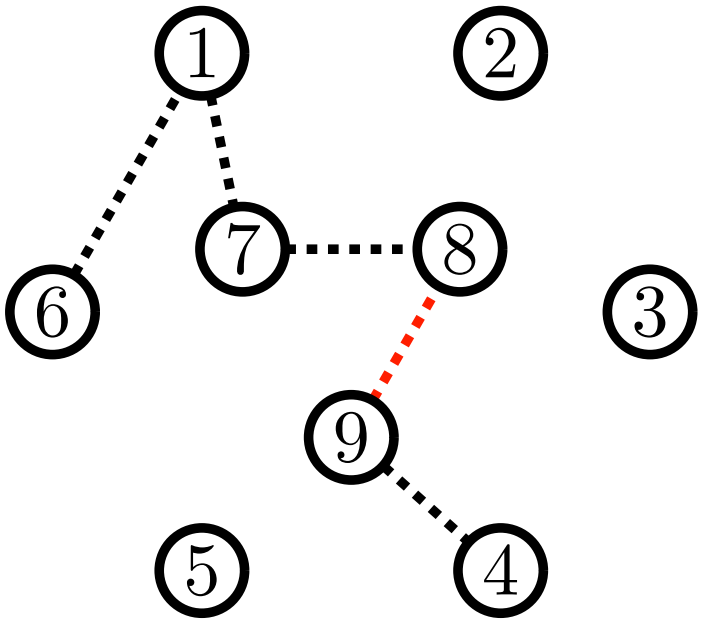

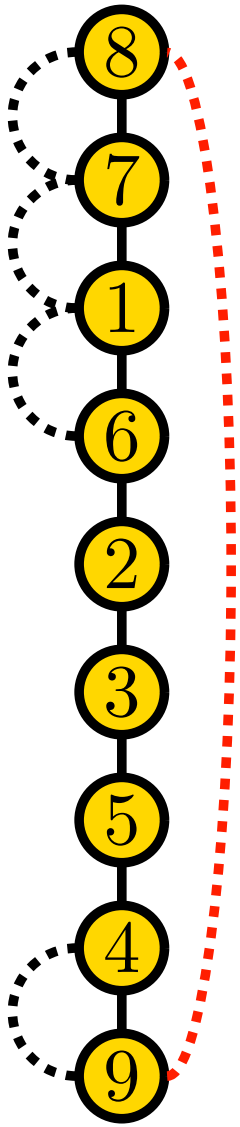

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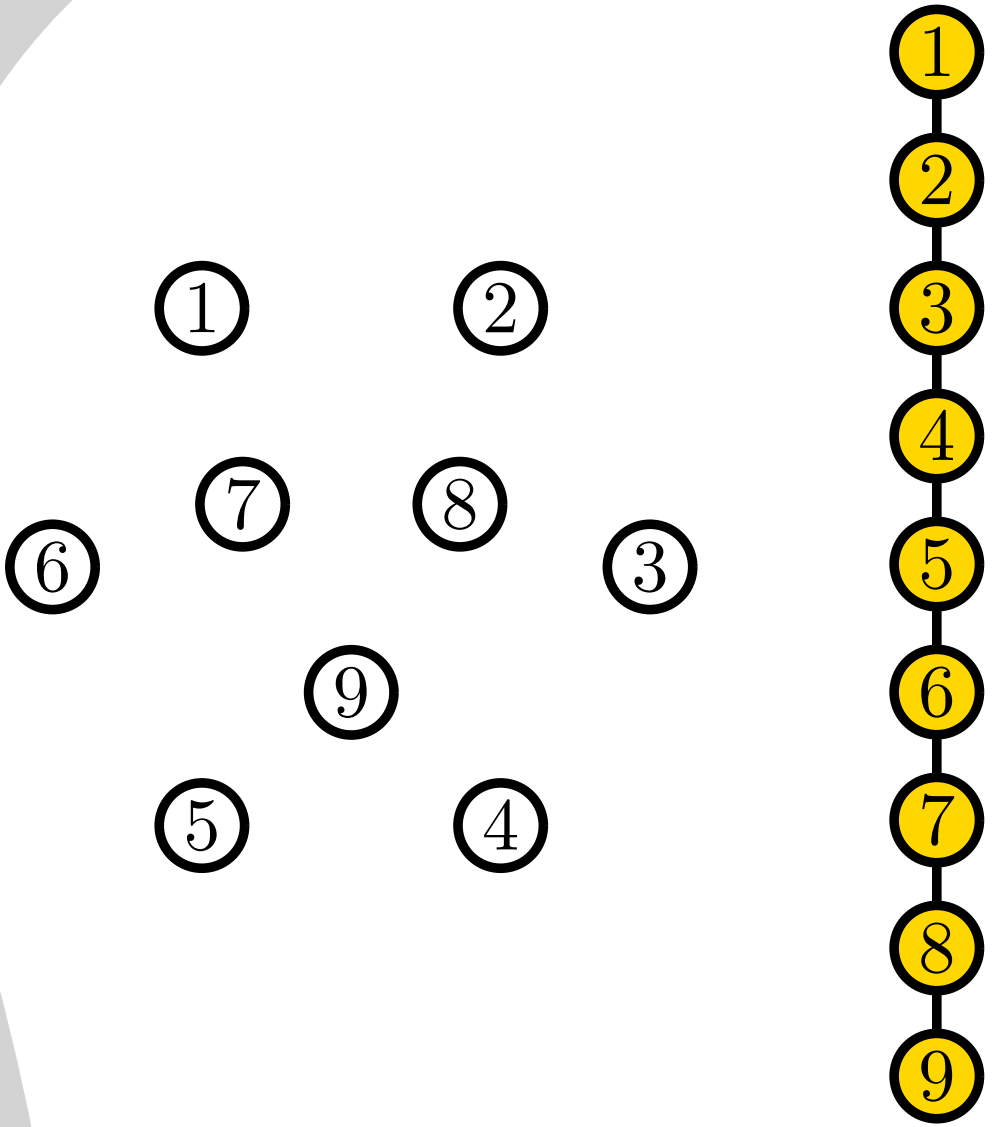
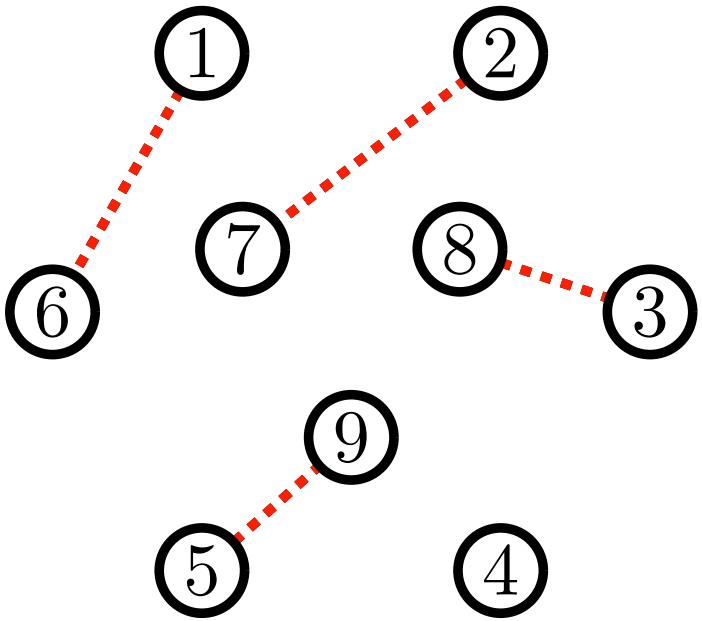
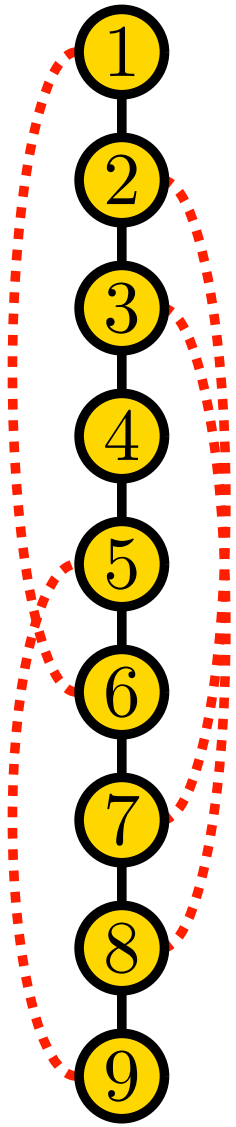


Lower Bound

The Strategy:

- Exploit Bad Edges
- Introduce Bad Matching





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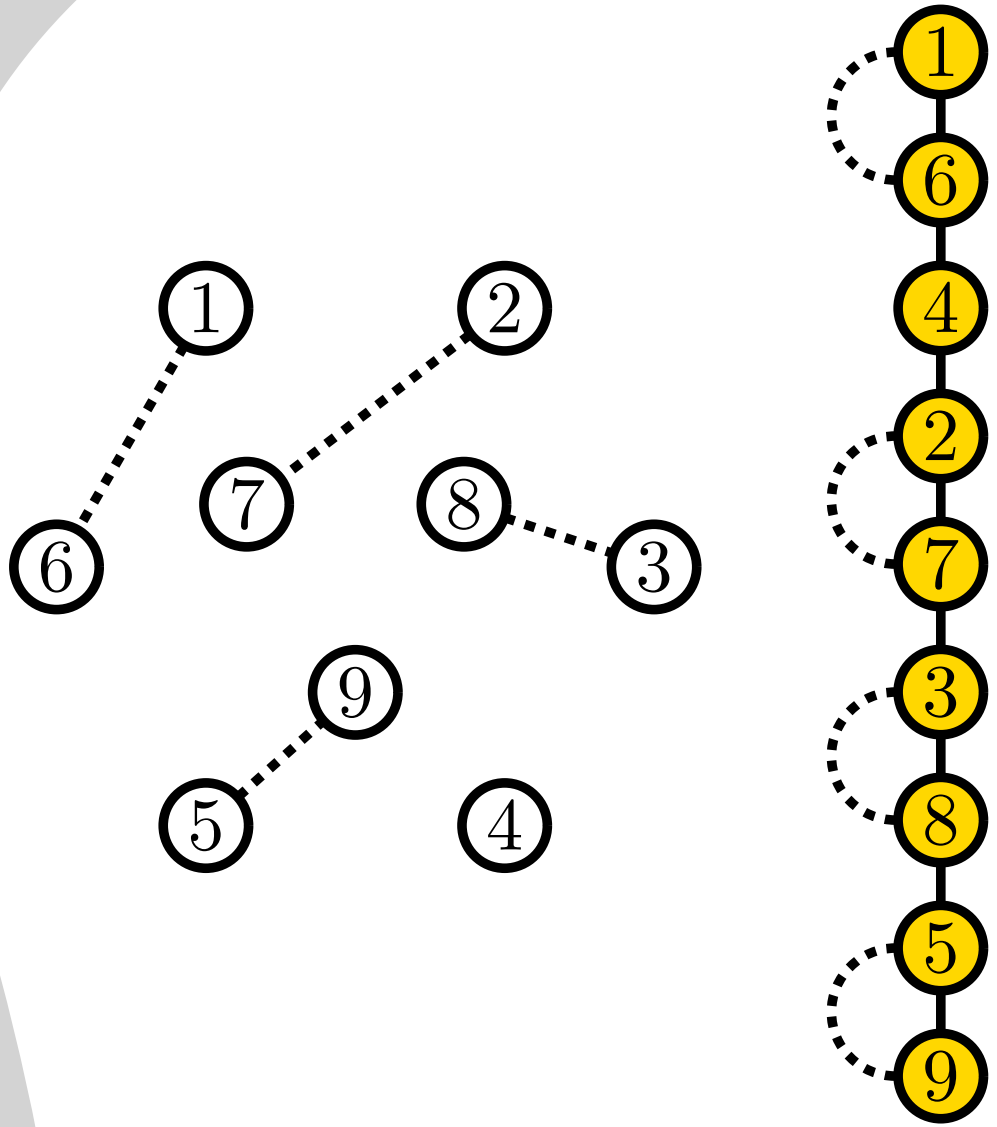

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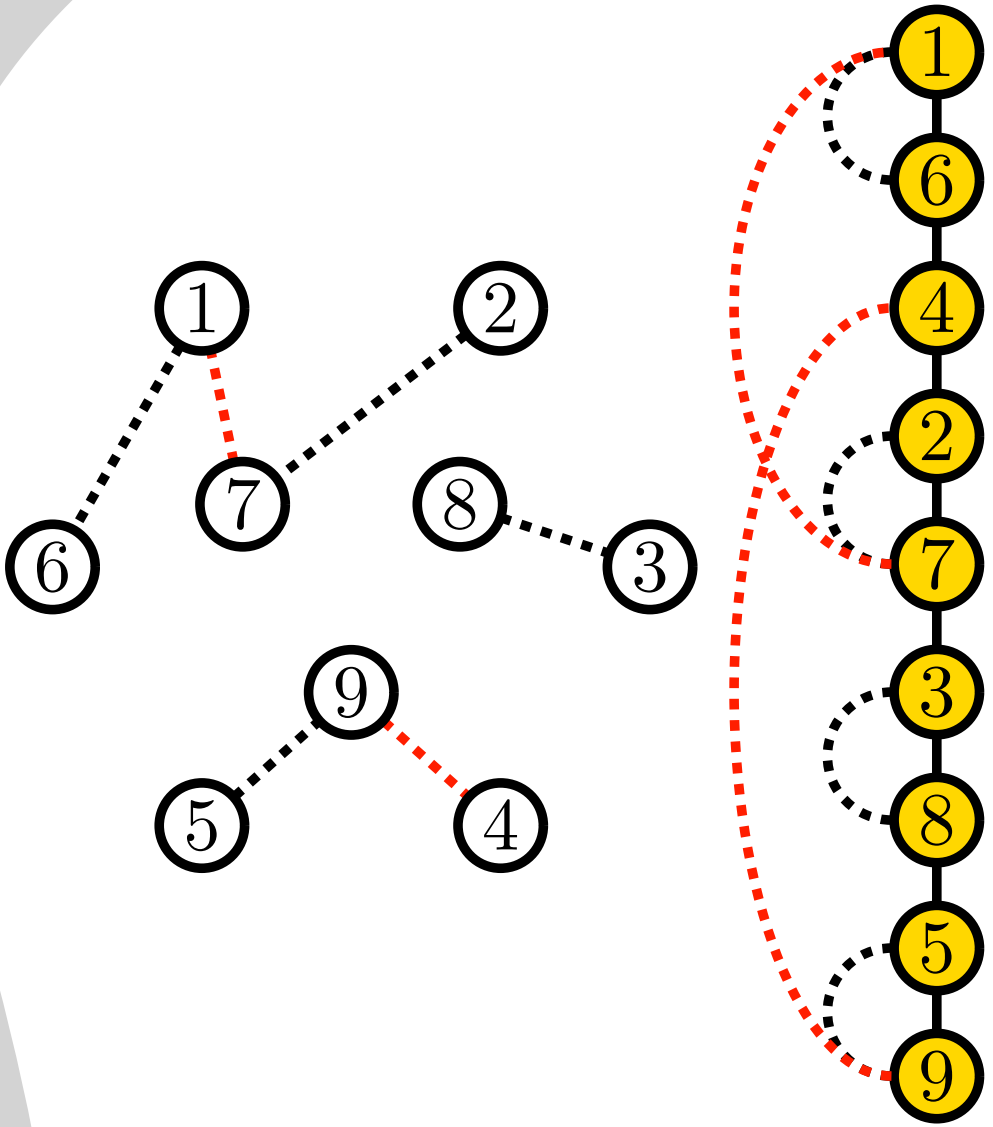

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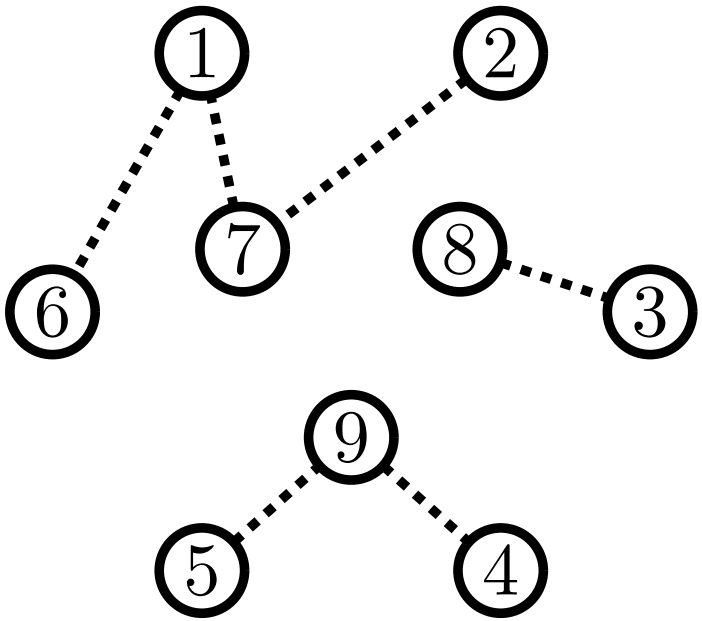

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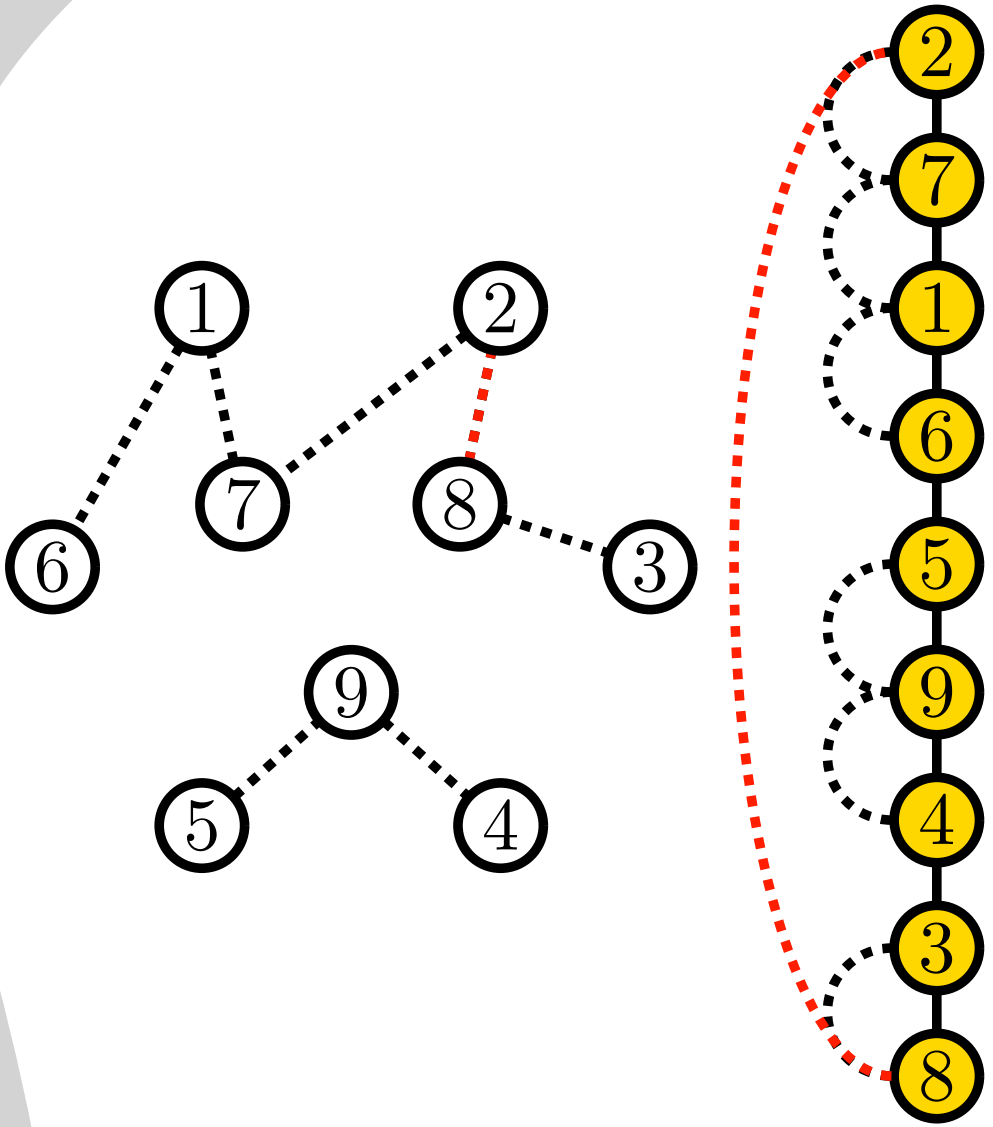

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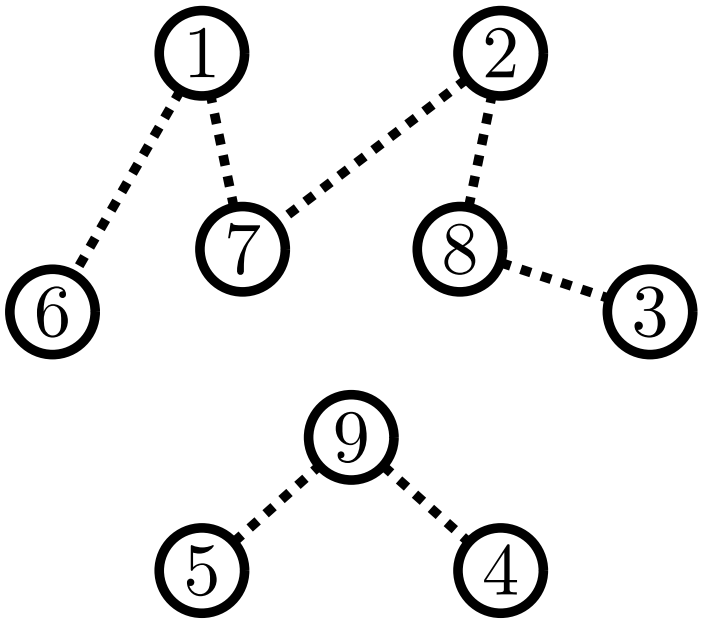

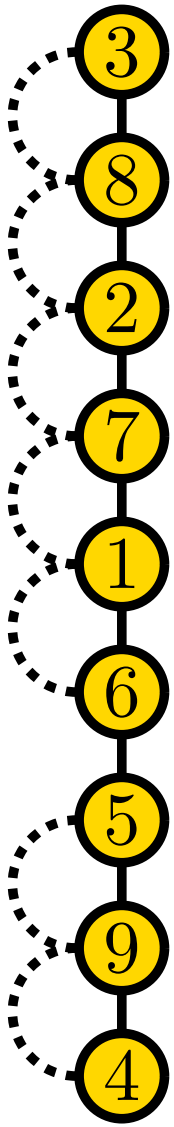

Lower Bound

The Strategy:

- Exploit Bad Edges
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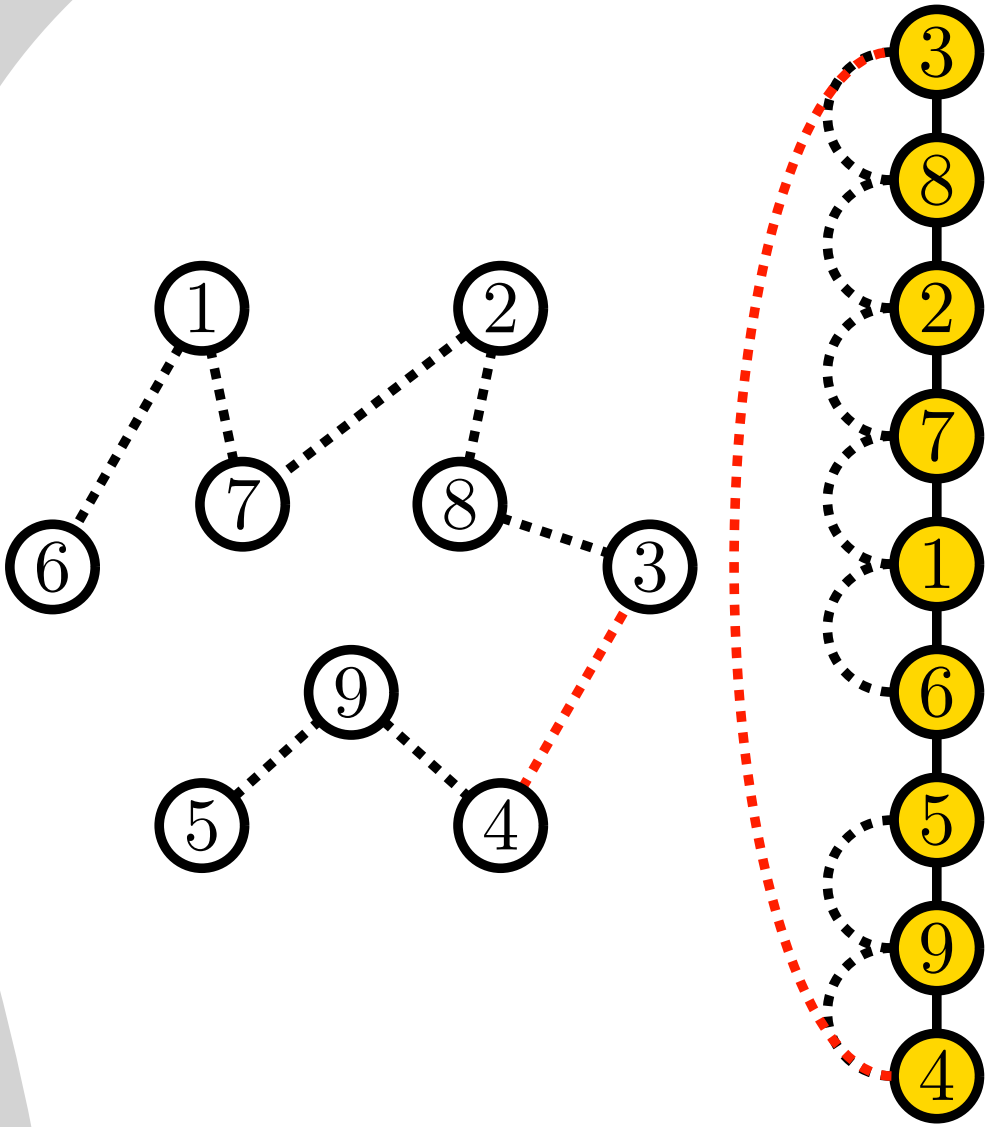

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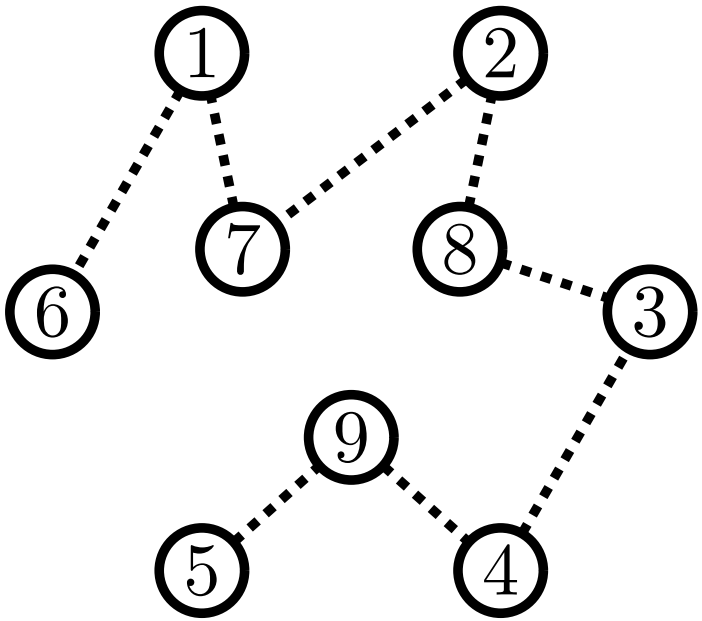

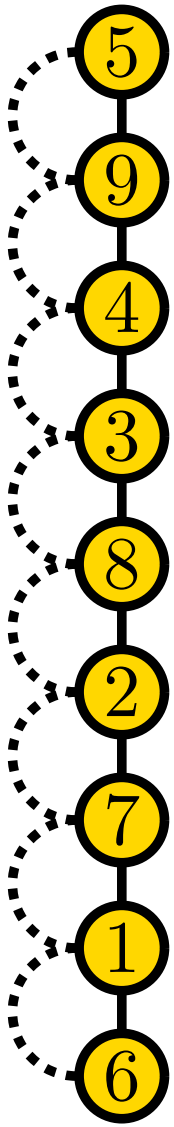

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The Strategy:

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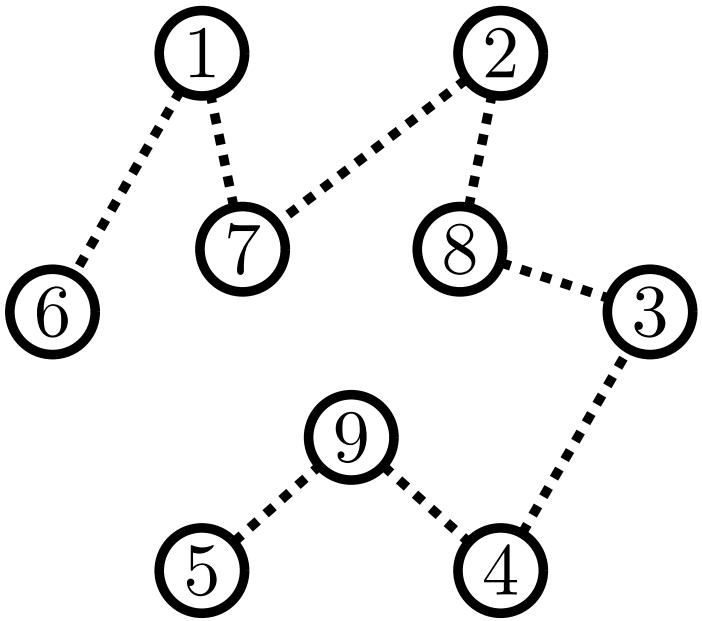

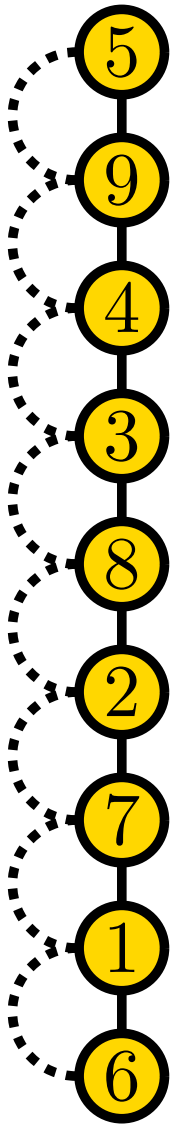

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The Strategy:

- Exploit Bad Edges
- Introduce Bad Matching
- Maintain List Graph

The Basic Math:

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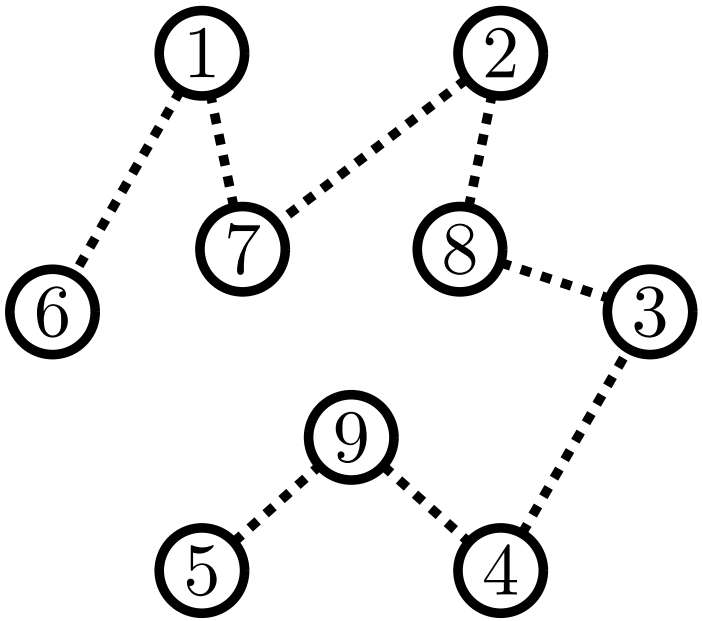
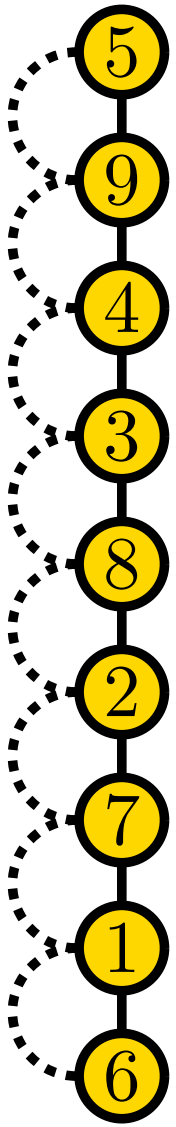
  $\rightarrow$ Competitive Ratio: $\Omega(\log n)$





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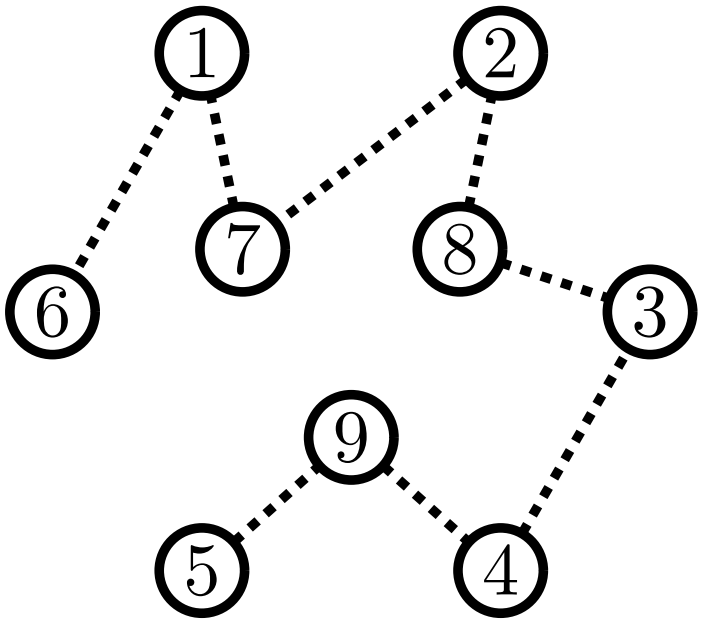

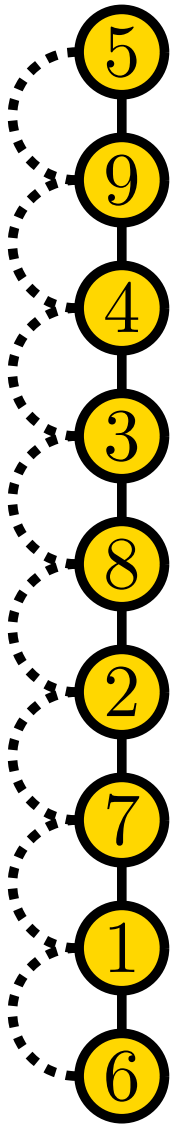

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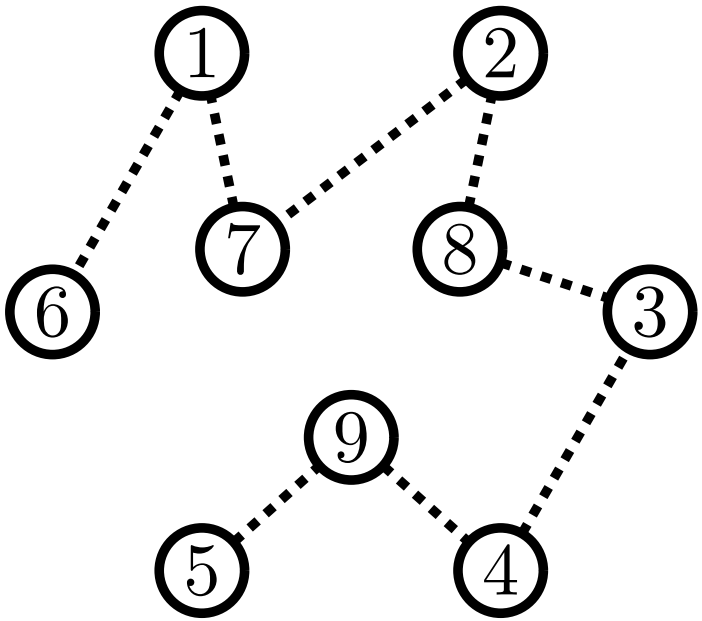
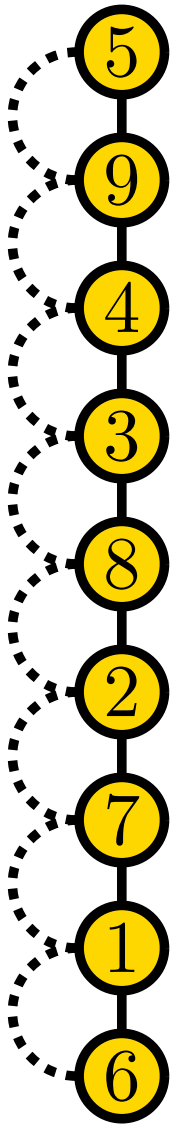
n^2 per Matching:

- Quantify Distortion of Matching





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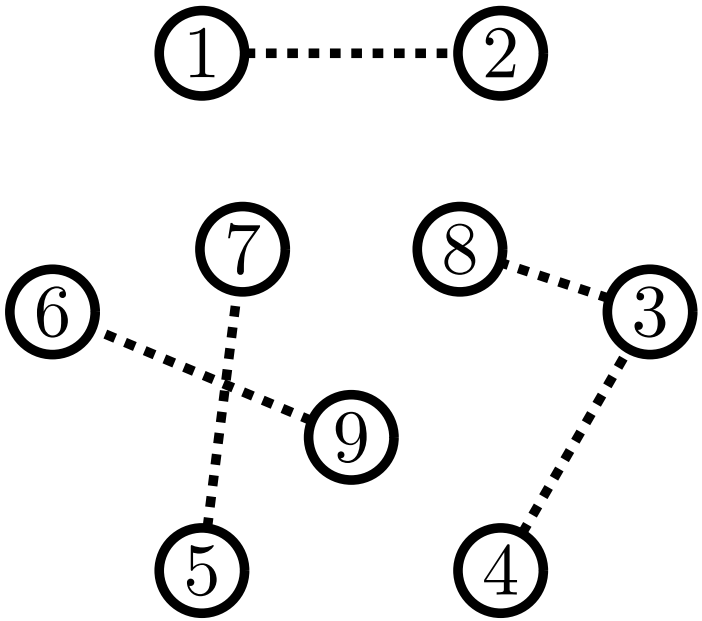

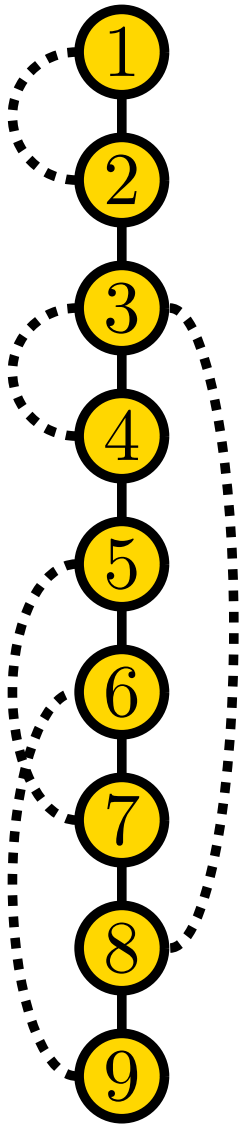

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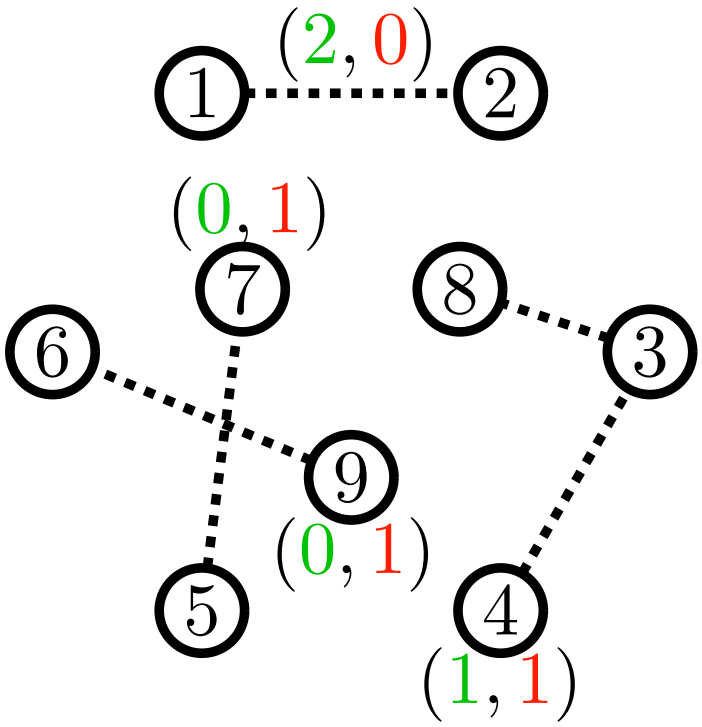

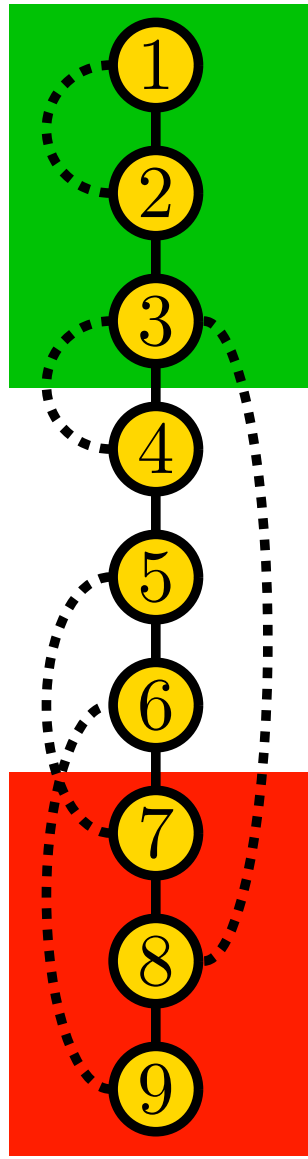

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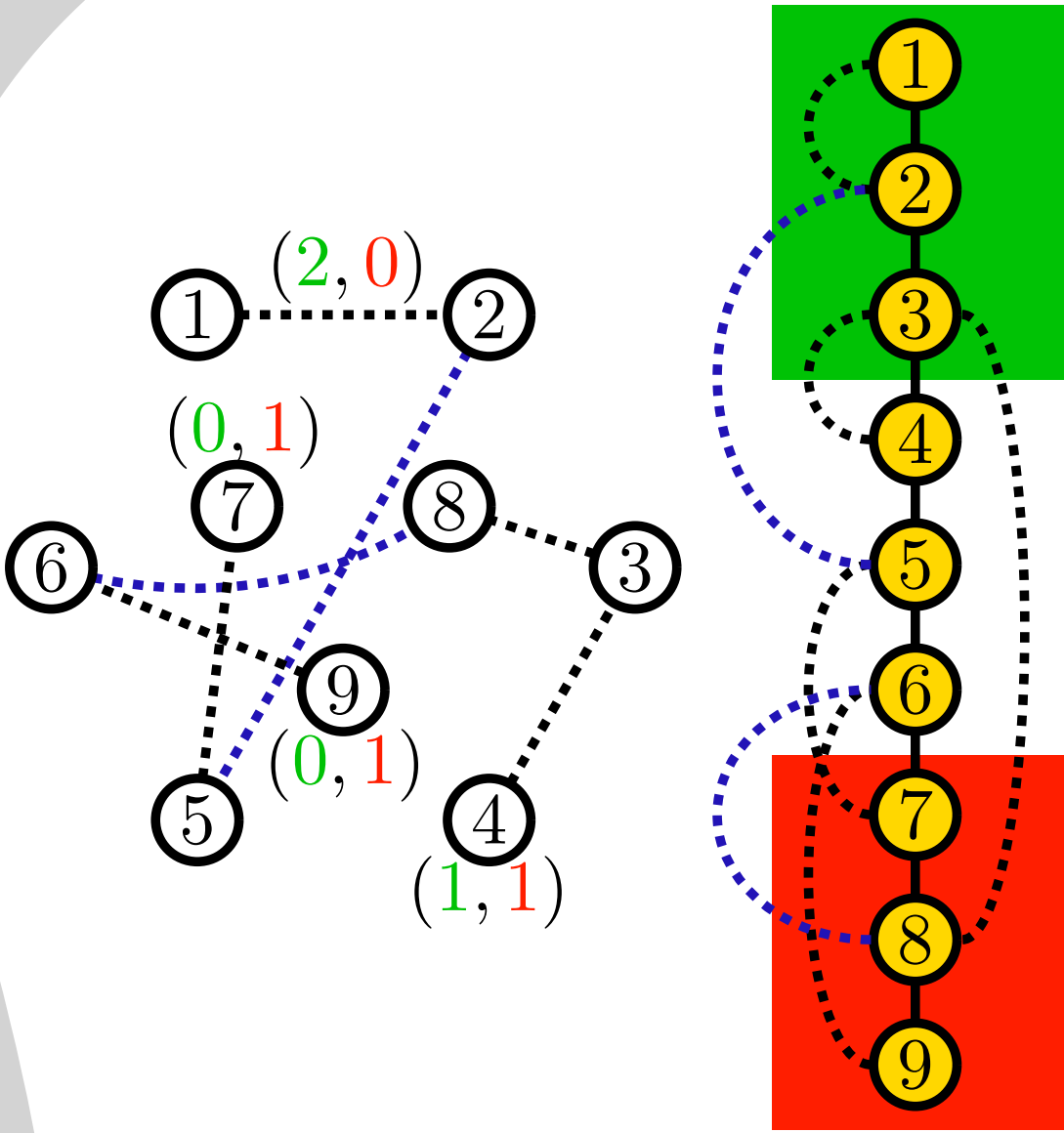

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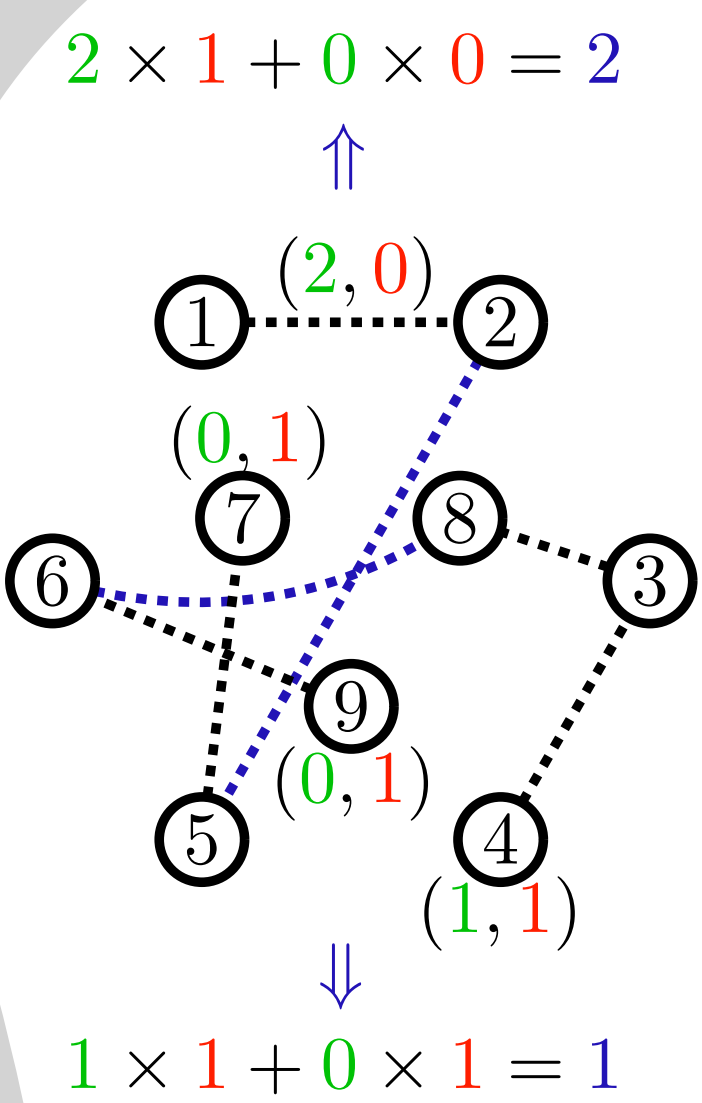

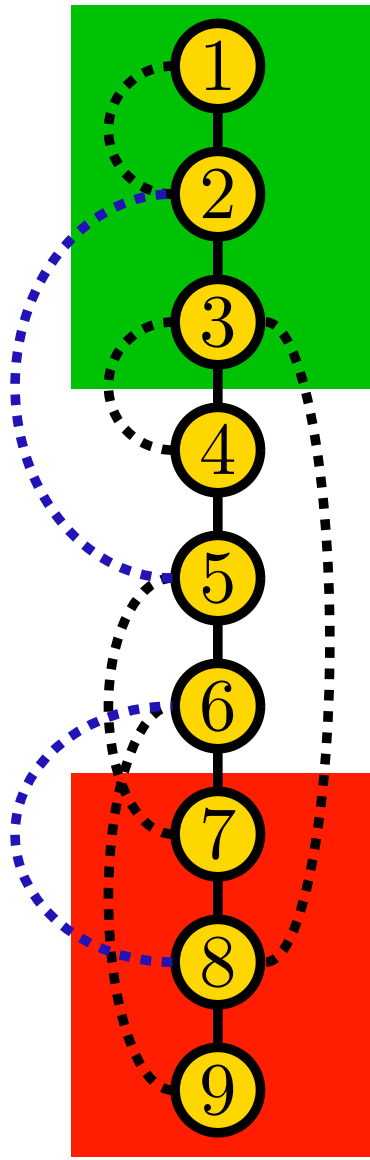

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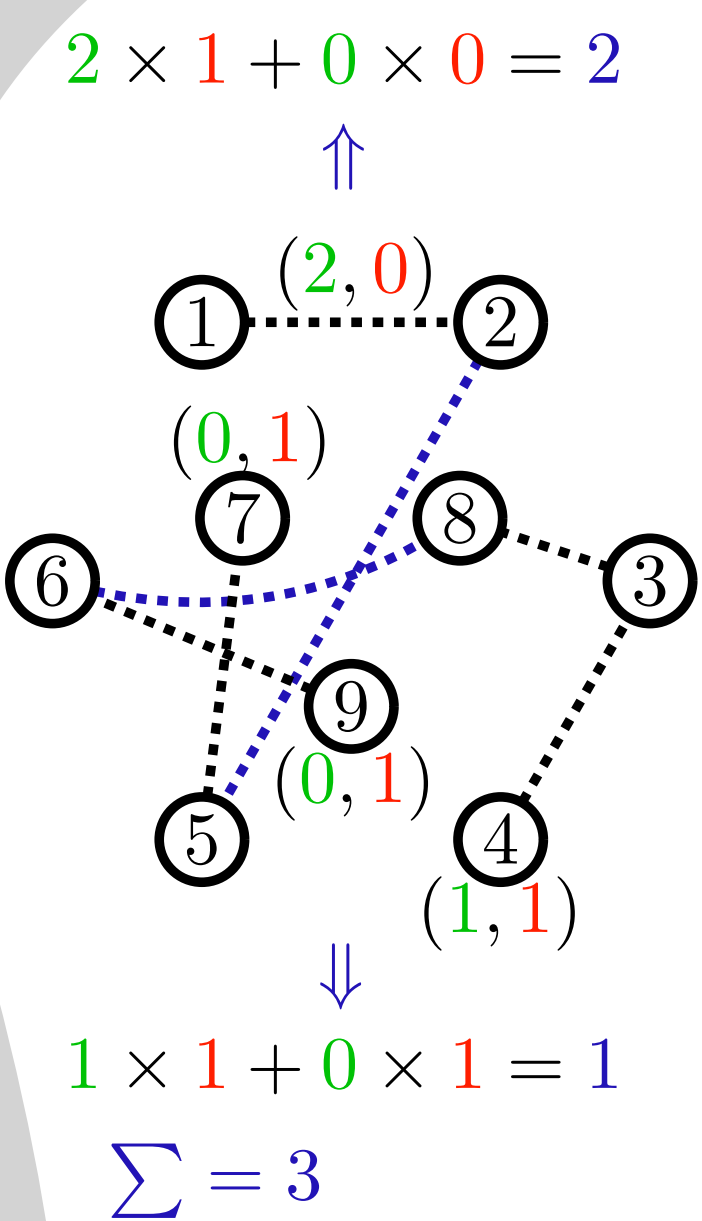

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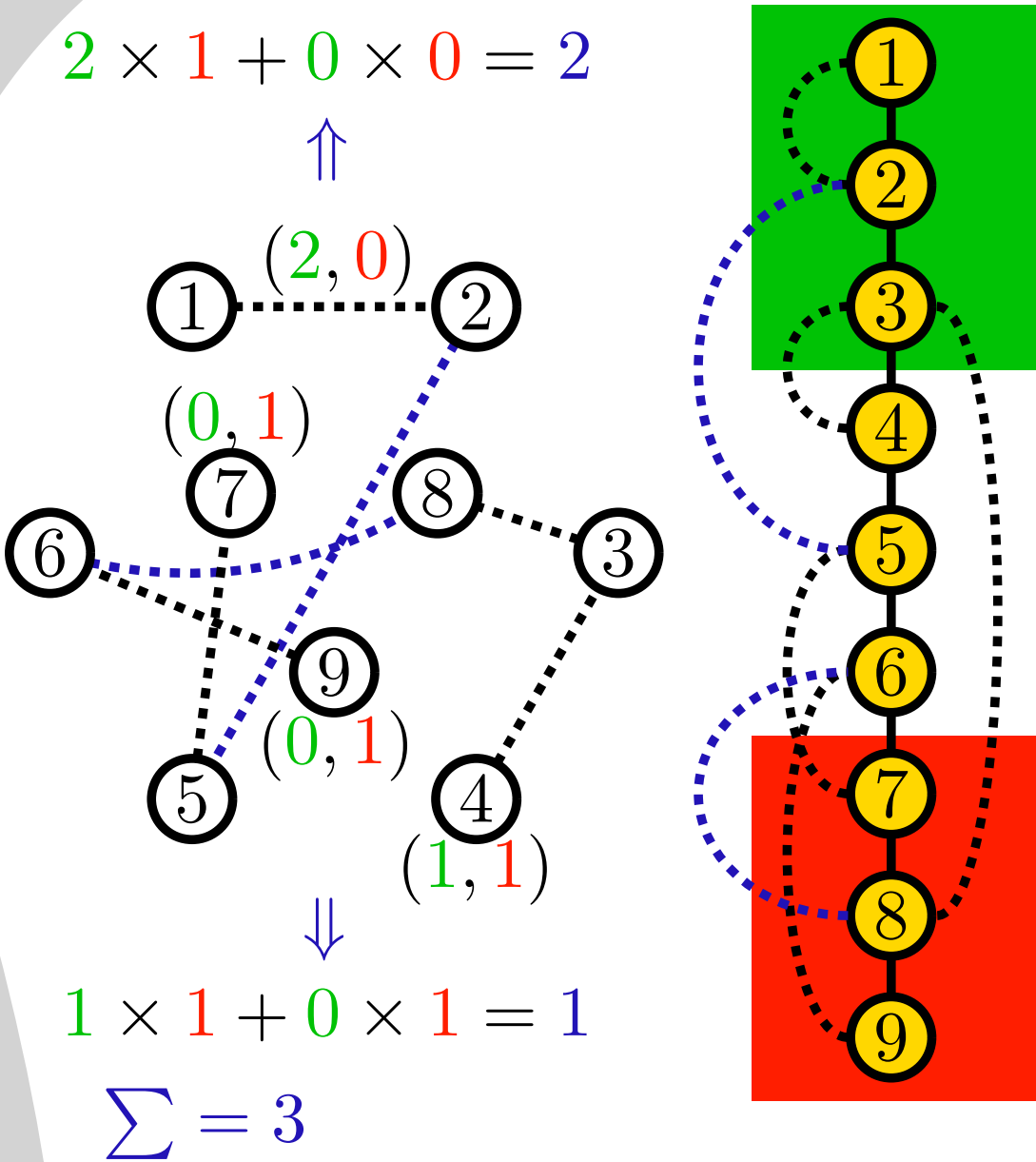
n^2 per Matching:

- Quantify Distortion of Matching
- Compute Sum of Distortion of all Matchings
- Avereage Distortion is Sufficient



Open Questions & Future Work

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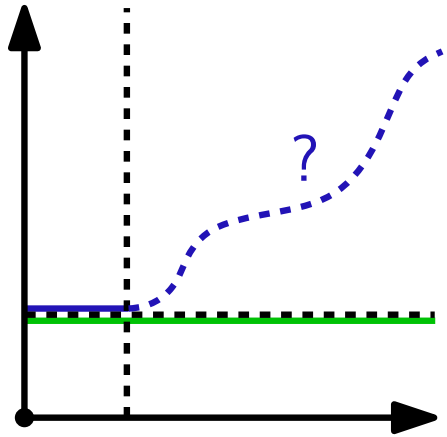
Better Bounds for Line Networks:

- Nontrivial Upper Bound?
- Better Lower Bound?
- Dynamic Offline Algorithms?



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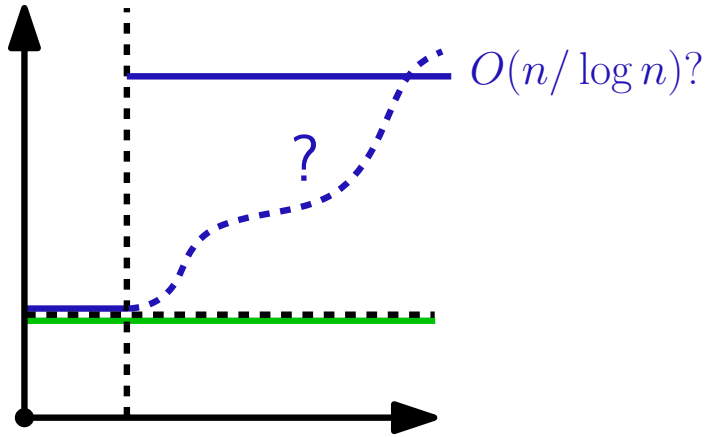

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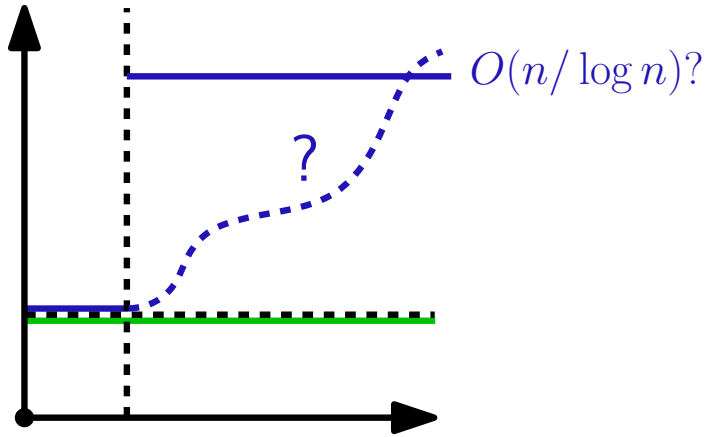

Different Networks:

- Binary (Splay) Trees
- No BST Property