A Robust Interference Model for Wireless Ad-Hoc Networks

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Abstract

Among the foremost goals of topology control in wireless ad-hoc networks is interference reduction. This paper presents a receiver-centric interference model featuring two main advantages over previous work. First, it reflects the fact that interference occurs at the intended receiver of a message. Second, the presented interference measure is robust with respect to addition or removal of single network nodes. Regarding both of these aspects our model intuitively corresponds to the behavior of interference in reality. Based on this interference model, we show that currently known topology control algorithms poorly reduce interference. Motivated by the observation that already one-dimensional network instances display the intricacy of the considered problem, we continue to focus on the so-called highway model. Setting out to analyze the special case of the exponential node chain, we eventually describe an algorithm guaranteeing to achieve a $\sqrt[4]{\Delta}$-approximation of the optimal connectivity-preserving topology in the general highway model.

1. Introduction

Wireless ad-hoc networks consist of mobile nodes equipped with, among other components, a processor, some memory, a wireless radio, and a power source. Due to physical constraints nodes are primarily powered by a weak battery.

Since consequently energy is the limiting factor for network lifetime, great efforts have been made to reduce node power consumption and thus extend network lifetime. One of the foremost approaches to achieve substantial energy conservation is by minimizing interference between network nodes. Confining interference lowers energy consumption by reducing the number of collisions and consequently packet retransmissions on the media access layer. The concept of *topology control* restricts interference by reducing the transmission power levels at the network nodes and cutting off long-range connections in a coordinated way. At the same time transmission power reduction has to proceed in such a way that the resulting topology preserves connectivity[1].

Even though interference reduction has always been one of the main motivations for topology control, most of the previous work addresses the interference issue implicitly by constructing topologies featuring sparseness or low node degree. However, [2] reveals that such an implicit notion of interference is not sufficient to reduce interference since message transmission can affect nodes even if they are not direct neighbors of the sending node in the resulting topology. Besides demonstrating the weakness of modeling interference implicitly, [2] introduces an explicit definition for interference in wireless networks.

The definition of interference suggested in [2] is problematic in two respects. First, it is based on the number of nodes affected by communication over a given link. In other words, interference is considered to be an issue at the sender instead of at the receiver, where message collisions actually prevent proper reception. It can therefore be argued that such sender-centric perspective hardly reflects real-world interference.

The second weakness of the model introduced in [2] is of more technical nature. According to its definition of interference, adding (or removing) a single node to a given network can dramatically influence the interference measure. In the network depicted in Figure 1, addition of the rightmost node to the cluster of roughly homogeneously distributed nodes entails the construction of a communication link covering all nodes in the network; accordingly—merely by introduction of one additional node—the interference value of the represented topology is pushed up from a small constant to the maximum possible value, that is the number of nodes in the network. This behavior contrasts to the intuition that a single additional node also represents but one additional packet source potentially causing collisions.

In summary, the concept of sender-centricity and the observed susceptibility of the proposed interference measure

1 The term “topology control” sometimes also refers to clustering and the computation of dominating sets. In this paper we exclusively consider topology control based on transmission power reduction.

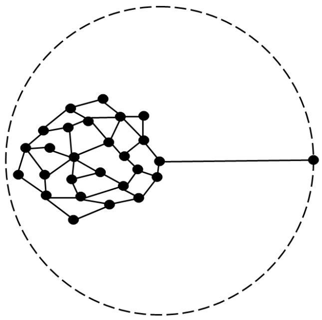

Figure 1. In the interference model presented in [2], addition of a single node increases interference from a small constant to the maximum possible value, the total number of network nodes.

to small changes in the network inevitably lead to the conclusion that the interference model introduced in [2] is questionable.

In contrast to this sender-centric interference definition, we explicitly consider interference at its point of impact, particularly at the receiver. Informally, the definition of interference considered in this paper is based on the natural question by how many other nodes a given network node can be disturbed. Our interference model is inspired by [4], where low interference topologies are analyzed in the context of data gathering, a generic application domain of sensor networks. In this paper we adapt this notion of interference to be suitable also in general ad-hoc networks. Technically, we consider arbitrary undirected networks as opposed to the directed data gathering trees studied in [4].

Interestingly, our interference definition not only reflects intuition due to its receiver-centricity. Moreover, this definition also results in a robust interference model in terms of measure increase due to the arrival of additional nodes in the network. Particularly, an additional node causes an interference increase of at most one at other nodes of the network. In clear contrast to the above sender-centric model, this corresponds to reality, where one added node contending for the shared medium constitutes only one additional possible collision source for nearby nodes in the network.

Interference reduction as such is meaningless—every node setting its transmission power to a minimum value trivially minimizes interference—without the formulation of additional requirements to be met by the resulting topology. In this paper we study the fundamental requirement that the considered topology control algorithms should preserve connectivity of the given network. For this requirement we show that most currently proposed topology control algorithms trying to reduce interference implicitly commit a substantial mistake—even by having every node connect to its nearest neighbor. Based on the intuition that already one-dimensional networks exhibit most of the complexity of finding minimum-interference topologies, we precisely anatomize networks restricted to one dimension—a model also known as the *highway* model. We first look at a particular network where distances between nodes increase exponentially from left to right. [11] introduces this network as a high interference example yielding interference $O(\Delta)$, where Δ is the maximum node degree. We show that it is intriguingly possible to achieve interference $O(\sqrt{\Delta})$ in our model for this network, which matches a lower bound also presented in this paper. Based on the insights gained thereby we then consider general highway instances where nodes can be distributed arbitrarily in one dimension. For the problem of finding a minimum-interference topology while maintaining connectivity we propose an approximation algorithm with approximation ratio $O(\sqrt[4]{\Delta})$.

The paper is organized as follows: After discussing related work in the following section, we introduce the model for this paper in Section 3. Section 4 focuses on the drawbacks of currently proposed topology control algorithms with respect to interference. In the subsequent section we consider the important case where nodes are distributed in one dimension by providing a lower bound for the interference in such networks and presenting an algorithm that matches this lower bound. Section 6 concludes the paper.

2. Related Work

In a very general way, topology control can be considered the task of—given a network communication graph—constructing a subgraph with certain desired properties. A first generation of topology control algorithms [6, 7, 14, 15], adopting structures from the field of computational geometry, focused on preserving energy-efficient paths or computing planar subgraphs for geometric routing [1, 8]. In a second wave of research, initiated by the CBTC algorithm [18], constructions were proposed which are based on local information and simultaneously reconcile several properties, such as planarity, the spanner property, or constant-bounded node degree [9, 10, 17]. Other approaches try to build on minimal assumptions about the capabilities of nodes and signal propagation characteristics [19].

If these contributions often mention interference reduction as one of the aims of topology control, this goal is stated to be achieved implicitly. In particular generating sparse constructions or topologies with low degree is commonly maintained to imply low interference.

An exception in this respect is formed by [11], introducing an explicit definition of interference and establishing trade-offs between the concepts of congestion, power consumption, and dilation. With [11] more attention is also being paid to the fact that—if nodes are capable of adapting their transmission power, an assumption already made in early work that can be considered originators of topology control considerations [5, 16]—interference ranges correlate with the length of communication links. More precisely the interference range of a link depends on the transmission power levels chosen by the two nodes communicating over the respective link.

Where [11] defines interference based on current network traffic, [2] introduces a traffic-independent notion of interference. Moreover, the latter work shows that the above statement that graph sparseness or small degree implies low interference is misleading. The interference model described in [2]—further analyzed in [12]—builds on the question of how many nodes are affected by communication over a given link. This sender-centric perspective can however be accused to be somewhat artificial and to poorly represent reality, interference occurring at the intended *receiver* of a message. Furthermore, as described in the introduction, this interference measure is susceptible to drastic changes even if single nodes are added to or removed from a network.

An attempt to correct for this deficiency was made in [4], which defines a receiver-centric concept of interference in the context of data-gathering structures in sensor networks. In this paper we go beyond [4] by defining and employing a suitable robust interference model for the analysis of topology control in ad-hoc networks in general.

3. Network and Interference Model

We model the wireless network with the well-known *Unit Disk Graph* (UDG) [3]. In a UDG $G = (V, E)$, there is an edge $\{u, v\} \in E$ iff the Euclidean distance between u and v is at most 1. That is, we assume all nodes to have the same limited transmission ranges. In the following, let Δ refer to the maximum node degree in G. In order to prevent already basic communication between neighboring nodes from becoming unacceptably cumbersome [13], it is required that a message sent over a link can be acknowledged by sending a corresponding message over the same link in the opposite direction. In other words, only *undirected* (symmetric) edges are considered.

We assume that each node can adjust its transmission power to any value between zero and its maximum transmission power level. The main goal of a *topology control* algorithm is then to compute a subgraph of the given network graph G that maintains connectivity by reducing transmission power levels of the nodes in V and thereby attempting to reduce interference and energy consumption.

Let N_u denote the set of all neighbors of a node $u \in V$ in the resulting topology. Then, each node u features a value r_u defined as the distance from u to its farthest neighbor. More precisely $r_u = max_{v \in N_u}\{|u, v|\}$, where $|u, v|$ denotes the Euclidean distance between nodes u and v. Since we assume the nodes to use omnidirectional antennas, $D(u, r_u)$ denotes the disk centered at u with radius r_u covering all nodes that are possibly affected by message transmission of u to one of its neighbors. Then the interference of a node v is defined as the number of other nodes that potentially affect message reception at node v:

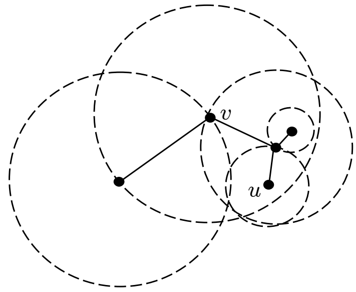

Figure 2. A sample topology consisting of five nodes with their corresponding interference radii (dashed circles). Node u experiences interference $I(u) = 2$ since it is covered not only by its direct neighbor but also by node v.

Definition 3.1. *Given a graph $G' = (V, E')$, the interference of a node $v \in V$ is defined as*

$$I(v) = |\{u | u \in V \setminus \{v\}, v \in D(u, r_u)\}|.$$

In other words, the interference I of a node v represents the number of nodes covering v with their disks induced by their transmission ranges set to a value as to reach their farthest neighbor in G'. Note that even though each node is also covered by its own disk, we do not consider this kind of self-interference. The node level interference defined so far is now extended to a graph interference measure as the maximum interference occurring in a graph:

Definition 3.2. *The interference of a graph $G' = (V, E')$ is defined as*

$$I(G') = \max_{v \in V} I(v).$$

Note that Δ, the maximum node degree of the given UDG $G = (V, E)$ is an upper bound for the interference of any subgraph G' of the given graph since in G each node is directly connected to all potentially interfering nodes. However, in arbitrary subgraphs of G the degree of a node only lower-bounds the interference of that node because a node can be covered by non-neighboring nodes (cf. Figure 2).

In this paper we study the combinatorial optimization problem of finding a resulting topology which maintains connectivity of the given network with minimum interference. Throughout the paper we only consider topologies consisting of a tree for each connected component of the given network since additional edges might unnecessarily increase interference.

4. Interference in Known Topologies

As motivated in the previous section, we restrict our considerations to resulting topologies consisting exclusively of symmetric links (edges).

To the best of our knowledge, all currently known topology control algorithms (with one exception, as explained later) constructing only symmetric connections have in common that every node establishes a link to at least its nearest neighbor. Technically, this means that these topologies contain the so-called *Nearest Neighbor Forest* as a subgraph. In this section, we show that this is already a substantial mistake, as thus interference becomes asymptotically incomparable with the interference-minimal topology.

Theorem 4.1. *Any algorithm containing the Nearest Neighbor Forest can have $\Omega(n)$ times larger interference than the interference of the optimum connected topology.*

Proof. Our proof uses a node distribution for which the Nearest Neighbor Forest yields interference $\Omega(n)$ while the optimum interference is in $\mathrm{O}(1)$. Note that this example has already been studied in [2], but for a different model.

Consider Figure 3: On the top, there is a horizontal chain of nodes h_i with exponentially growing distances, that is, the distance between nodes h_i and h_{i+1} is 2^i. Each of these nodes h_i has a corresponding node v_i vertically displaced by a little more than h_i's distance to its left neighbor, that is, $d_i > 2^{i-1}$ holds, where d_i is the distance between h_i and v_i. Note that the nodes v_i also form a (diagonal) exponential node chain. Finally, between two of these diagonal nodes v_{i-1} and v_i, an additional helper node t_i is placed such that $|h_i, t_i| > |h_i, v_i|$.

The Nearest Neighbor Forest for this node distribution—assuming that the transmission radius of each node can be chosen sufficiently large—is shown in Figure 4. In order to calculate the interference, we first observe that an edge from h_i to h_{i+1} covers all nodes to the left, that is, all nodes h_j for $j < i$. In particular, the leftmost node h_0 on the horizontal chain is covered by all nodes h_i with $i > 0$. As roughly one third of all nodes are part of the horizontally connected exponential chain, h_0 is covered by at least $\Omega(n)$ nodes.

The optimal tree on the other hand does not connect the horizontal node chain, as depicted in Figure 5. In particular, it is easy to see that the resulting graph has constant interference. □

Finally, it has to be mentioned that, as a notable exception, the topology control algorithms presented in [2] do not necessarily include the Nearest Neighbor Forest. Unfortunately however, it can be shown that also those algorithms perform badly for our interference model.

5. Analysis of the Highway Model

In this section we study interference for the *highway* model in which the node distribution is restricted to one dimension. After analyzing an important artificially constructed problem instance, we provide a lower bound for interference of general problem instances in the highway model as well as an asymptotically optimal algorithm matching this bound. Finally, an approximation algorithm is presented.

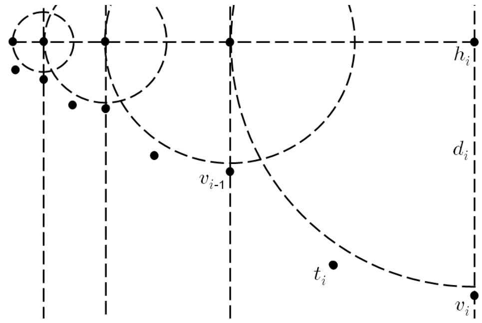

Figure 3. Two exponential node chains.

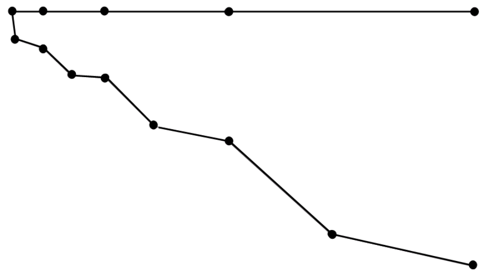

Figure 4. The Nearest Neighbor Forest yields interference $\Omega(n)$.

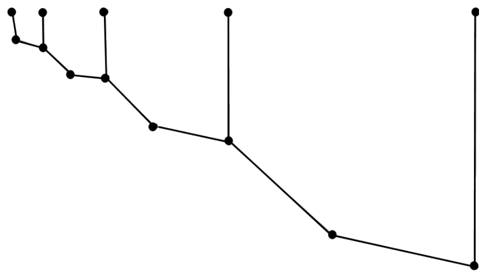

Figure 5. Optimal tree with constant interference.

5.1. The Exponential Node Chain

How can n nodes arbitrarily distributed in one dimension connect to each other minimizing interference while maintaining connectivity? [11] introduces an instance which seems to yield inherently high interference: the so called *exponential node chain* is a one-dimensional graph $G = (V, E)$ where the distance between two consecutive nodes grows exponentially from left to right as depicted in Figure 6. The distance between two nodes v_i and v_{i+1} in V is thus 2^i. Throughout the consideration of the exponential node chain we assume that the distance between the leftmost and the rightmost node is not greater than 1: Each node can potentially connect to all other nodes in V and therefore $\Delta = n - 1$, where $n = |V|$. The nodes are termed *linearly connected* if each node—except for the leftmost and

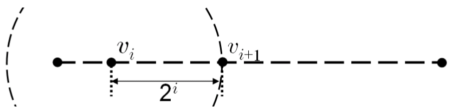

Figure 6. In the exponential node chain, the distance between two consecutive nodes grows exponentially from left to right.

the rightmost—maintains an edge to its nearest neighbor to the left and to the right, that is, node v_i is connected to node v_{i+1} for all $i = 1, \ldots, n-1$ in the resulting topology. In addition to the disks $D(v_i, r_{v_i})$ for each node $v_i \in V$, Figure 7 depicts their interference values $I(v_i)$. Since all but the disk of the rightmost node cover v_1, interference at the leftmost node is $n - 2 \in \Omega(n)$ and consequently also interference of the linearly connected exponential node chain is in $\Omega(n)$.

As we show in the following, the exponential node chain can surprisingly be connected in a significantly better way.

According to the construction of the exponential node chain, only nodes connecting to at least one node to their right increase v_1's interference. We call such a node a *hub* and define it as follows:

Definition 5.1. *Given a connected topology for the exponential node chain $G = (V, E)$. A node $v_i \in V$ is defined to be a* hub *in G iff there exists an edge (v_i, v_j) with $j > i$.*

The following algorithm $\mathcal{A}_{exp}$ constructs a topology for the exponential node chain G that yields interference $\mathrm{O}(\sqrt{n})$. The algorithm starts with a graph $G_{exp} = (V, E_{exp})$, where V is the set of nodes in the exponential node chain and E_{exp} is initially the empty set. Following the scan-line principle, $\mathcal{A}_{exp}$ processes all nodes in the order of their occurrence from left to right. Initially, the leftmost node is set to be the current hub h. Then, for each node v_i $\mathcal{A}_{exp}$ inserts an edge $\{h, v_i\}$ into E_{exp}. This is repeated until $I(G_{exp})$ increases due to the addition of such an edge. Now node v_i becomes the current hub and subsequent nodes are connected to v_i as long as the overall interference $I(G_{exp})$ does not increase. Figure 8 depicts the resulting topology if $\mathcal{A}_{exp}$ is applied to the exponential node chain. The exponential node chain is thereby depicted in a logarithmic scale. For clarity of representation, edges in E_{exp} are depicted as arcs. In addition, Figure 8 shows the individual interference values at each node.

In the following we show that $\mathcal{A}_{exp}$ reduces interference in the exponential node chain.

Theorem 5.1. *Given the exponential node chain G, applying Algorithm $\mathcal{A}_{exp}$ results in a connected topology with interference $I(G_{exp}) \in \mathrm{O}(\sqrt{n})$.*

Proof. The topology resulting to application of $\mathcal{A}_{exp}$ shows a clear structure (cf. Figure 8). Each hub, not taking into account the first two, is connected to one more node to its right than its predecessor hub to the left. This follows from the fact that if the current topology leads to interference $I(G_{exp}) = I$ immediately after the determination of a new hub, this hub can be connected to $I - 1$ nodes to its right until $I(G_{exp})$ is again increased by one. Therefore the minimum number of nodes n required in an exponential node chain, such that interference $I(G_{exp}) = I$ is obtained, results in

$$n = \sum_{i=1}^{I-1} i + 2 = \frac{1}{2}I^2 - \frac{1}{2}I + 2.$$

By solving for I, with $n \geq 2$, we have

$$I = \left\lfloor \frac{\sqrt{8n - 15} + 1}{2} \right\rfloor \in \mathrm{O}(\sqrt{n}).$$

□

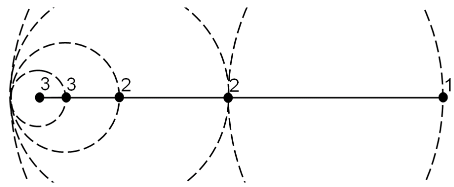

Figure 7. Connecting the exponential node chain linearly yields interference of $n - 2$ at the leftmost node since each node connected to the right covers all nodes to its left. The nodes are labelled according to their experienced interference.

This is an intriguing result since we show in the sequel that $\sqrt{n}$ is a lower bound for the interference of the exponential node chain.

In the following we now show that there exist network instances where every possible topology exhibits interference at least $\sqrt{n}$. We therefore again consider the exponential node chain introduced in Section 5.1 with all n nodes located within distance one.

Theorem 5.2. *Given an exponential node chain $G = (V, E)$ with $n = |V|$, $\sqrt{n}$ is a lower bound for the interference $I(G)$.*

Proof. Let H denote the set of hubs (cf. Definition 5.1) in G and S the nodes in $G \setminus H$. In order to prove the theorem, we state two properties for $I(G)$ in the exponential node chain G. First, it holds that $I(G)$ is at least $|H| - 1$, since the leftmost node is interfered with by exactly all hubs except itself (Property 1). On the other hand, $I(G)$ is at least the maximum degree of the resulting topology (Property 2). This holds since a node with maximum degree is covered by at least all disks of its neighboring nodes. We assume for the sake of contradiction that there exists a connected graph that yields interference less than $\sqrt{n}$ for the exponential node chain G. In other words, the degree of any node is

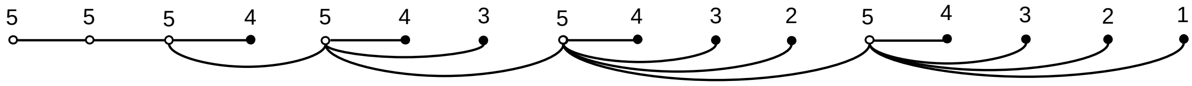

Figure 8. The interference of the exponential node chain—shown in a logarithmic scale—is bounded by $O(\sqrt{n})$ by the topology control algorithm $\mathcal{A}_{exp}$. Only hubs (hollow points) interfere with the leftmost node. For clarity of representation edges are depicted as arcs.

required to be at most $\sqrt{n}-1$, and the number of hubs must not exceed $\sqrt{n}$, including the leftmost node. By the definition of H and S, each node in the graph is either in H or in S and therefore $|H| + |S| = n$ holds. Due to Property 1, it follows that $|H| \leq \sqrt{n}$. Without loss of generality we assume that the hubs are linearly connected among themselves in order to guarantee connectivity of the graph. Consequently, with Property 2, each hub can connect to at most $\sqrt{n} - 3$ nodes in S (the leftmost and the rightmost hub, respectively, to $\sqrt{n} - 2$). By the definition of a hub, nodes in S are only connected to hubs and not among themselves. Therefore we obtain $|S| \leq \sqrt{n}\,(\sqrt{n} - 3) + 2$. Consequently, $|H| + |S|$ results in $n - 2\sqrt{n} + 2$, which is less than n for $n \geq 2$ and thus leads to a contradiction. □

From Theorems 5.1 and 5.2 it follows that Algorithm $\mathcal{A}_{exp}$ from Section 5.1 is asymptotically optimal in terms of interference in the exponential node chain.

5.2. The General Highway Model

We have considered an important artificially constructed instance in the highway model in Section 5.1, yielding a lower bound for the interference in arbitrary network graphs. In this section we do not restrict ourselves to particular network instances but consider arbitrary distributed nodes in one dimension.

The question arises if there are instances in the highway model that are asymptotically worse than the exponential node chain, that is, where a minimum-interference topology exceeds $\Omega(\sqrt{\Delta})$. We answer this question in the negative by introducing Algorithm $\mathcal{A}_{gen}$, which yields interference in $O(\sqrt{\Delta})$ for *any* given node distribution.

In a first step, the algorithm determines Δ of the given Unit Disk Graph $G = (V, E)$ and partitions "the highway" into segments of unit length 1. That is, within such a segment each node can potentially connect to every other node in the segment.

In a second step, $\mathcal{A}_{gen}$ considers each segment independently as follows: Starting with the leftmost node of the segment, every $\lceil\sqrt{\Delta}\rceil$-th node (according to their appearance from left to right) becomes a *hub*. A hub is thereby redefined along the lines of Definition 5.1 as a node that has more than one neighboring node, in contrast to *regular* nodes, which are connected to exactly one hub. In order to avoid boundary effects, the rightmost node of each segment is also considered a hub. Then, Algorithm $\mathcal{A}_{gen}$ connects the hubs of a segment linearly. That is, each hub, except the leftmost and the rightmost one, establishes an edge to its nearest hub to the left and to the right. Two consecutive hubs define an *interval*. $\mathcal{A}_{gen}$ connects all regular nodes in a particular interval to their nearest hub—ties are broken arbitrarily. Figure 9 depicts one segment of an example instance after the application of $\mathcal{A}_{gen}$. Again, edges are thereby depicted as arcs. The nodes within a segment clearly form one connected component.

Finally, Algorithm $\mathcal{A}_{gen}$ connects two adjacent segments by connecting the rightmost node of the left segment with the leftmost node of the right segment. This yields a connected topology if the corresponding unit disk graph is also connected. Note that using this construction, the hubs may have a comparatively high transmission range (smaller than one unit though). However, the interference range of regular nodes is restricted to their corresponding intervals. This is due to the fact that nodes are connected to their nearest hub only, which determines their transmission ranges.

To prove that the resulting topology of $\mathcal{A}_{gen}$ yields $O(\sqrt{\Delta})$ interference, we introduce an additional lemma, which shows that the interference of a node caused by other nodes in the same segment constructed by $\mathcal{A}_{gen}$ is in $O(\sqrt{\Delta})$.

Lemma 5.3. *Each node in a segment σ of Algorithm $\mathcal{A}_{gen}$ experiences at most $O(\sqrt{\Delta})$ interference in the resulting topology of $\mathcal{A}_{gen}$ caused by nodes in σ.*

Proof. By definition of a segment, Δ is an upper bound on the number of nodes in the segment. Algorithm $\mathcal{A}_{gen}$ nominates only every $\lceil\sqrt{\Delta}\rceil$-th node a hub. Thus, the number of hubs in σ is upper-bounded by $\Delta/\lceil\sqrt{\Delta}\rceil \in O(\sqrt{\Delta})$. Let hub h_l delimit the interval of a regular node v to the left, and hub h_r to the right, respectively. Furthermore, we can assume without loss of generality that $|h_l, v| < |v, h_r|$. Therefore, $\mathcal{A}_{gen}$ establishes a connection between h_l and v. Because this is the only connection of v it follows that $r_v = |h_l, v|$. Consequently, a regular node only interferes with nodes in the same interval. Since a node v is in at most two intervals—hubs are in two intervals—with at most $\lceil\sqrt{\Delta}\rceil$ nodes, v exhibits interference of at most $O(\sqrt{\Delta})$ regular nodes. Furthermore v is interfered with at most $O(\sqrt{\Delta})$ hubs. □

With Lemma 5.3 we are ready to prove that the topology constructed by $\mathcal{A}_{gen}$ results in $O(\sqrt{\Delta})$ interference.

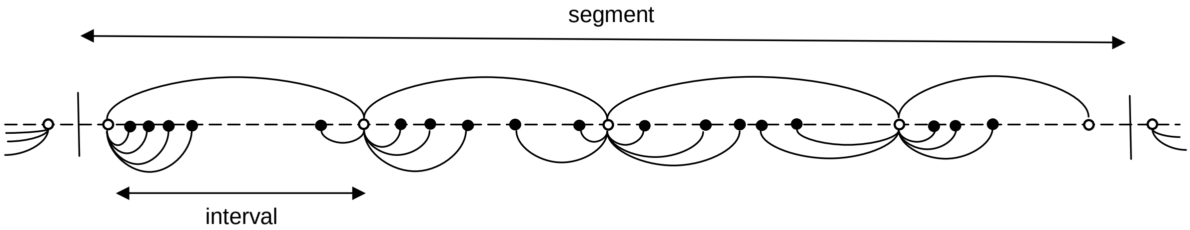

Figure 9. $\mathcal{A}_{gen}$ partitions the highway into segments of length 1. In each segment, every $\lceil\sqrt{\Delta}\rceil$-th node becomes a hub (hollow points). While the hubs are connected linearly, each of the remaining nodes in the interval between two hubs is connected to its nearest hub.

Theorem 5.4. *The resulting topology constructed by Algorithm $\mathcal{A}_{gen}$ from a given graph $G = (V, E)$ yields interference $\mathrm{O}(\sqrt{\Delta})$.*

Proof. By Lemma 5.3, the interference of a detached segment constructed by $\mathcal{A}_{gen}$ is bounded by $\mathrm{O}(\sqrt{\Delta})$. However, interference at node v in segment σ depends also on nodes in the adjacent segments of σ, referred to as σ_l for the segment to the left of σ and σ_r for the segment to the right, respectively. Nodes in other segments do not interfere with v as the length of a segment is chosen according to the maximum transmission range and thus the interference range of a node is limited to two adjacent segments. We know that at most $\mathrm{O}(\sqrt{\Delta})$ nodes of σ interfere v. On the other hand, by Lemma 5.3, the rightmost node v' of σ_l is also covered by at most $\mathrm{O}(\sqrt{\Delta})$ disks of nodes in σ_l. This implies that at most $\mathrm{O}(\sqrt{\Delta})$ nodes of σ_l interfere with v since all nodes interfering with v must also cover v' with their disks. By symmetry, the same holds for segment σ_r. Consequently, $\mathcal{A}_{gen}$ results in interference at most three times the interference of an individual segment at each node, which is in $\mathrm{O}(\sqrt{\Delta})$. □

5.3. Approximation Algorithm

In contrast to $\mathcal{A}_{gen}$, achieving interference in $\mathrm{O}(\sqrt{\Delta})$ for any network instance, this section introduces an algorithm that approximates the optimum for the given network instance. Particularly it yields interference at most a factor in $\mathrm{O}(\sqrt[4]{\Delta})$ times the interference value resulting from an interference-minimal connectivity-preserving topology.

Algorithm $\mathcal{A}_{gen}$ introduced in Section 5.2 is in a sense designed for the *worst-case*. Consider for example an instance where the distances between consecutive nodes are identical. Connecting these nodes linearly, that is, connecting each node to its nearest neighbor in each direction, yields constant interference. Algorithm $\mathcal{A}_{gen}$ however constructs a topology resulting in $\mathrm{O}(\sqrt{\Delta})$ interference since a hub connects to one half of the nodes in its corresponding interval for this instance and an interval contains $\lceil\sqrt{\Delta}\rceil$ nodes. Based on this observation, we introduce Algorithm $\mathcal{A}_{apx}$, a hybrid algorithm which detects high interference instances and applies $\mathcal{A}_{gen}$, or otherwise connects the nodes linearly.

In the following, we first present a suitable criterion to identify "high interference" instances. Given a network graph $G = (V, E)$ in the highway model, let the graph $G_{lin} = (V, E_{lin})$ denote the graph where all nodes in V are linearly connected. In order to result in high interference at a node v in G_{lin}, it is required that many nodes cover v with their corresponding disks. However, with increasing distance to v these nodes need increasing distances to their nearest neighbors in the opposite direction of v to interfere with the latter. This leads to an exponential characteristic of these nodes since the edges in E_{lin} accounting for the interference at v form a fragmented exponential node chain. Consequently, the *critical nodes* of v are defined as follows:

Definition 5.2. *Given a linearly connected graph $G_{lin} = (V, E_{lin})$. The critical node set of a node v is defined as*

$$C_v = \{u | u \neq v, |u, w| \geq |u, v|, \{u, w\} \in E_{lin}\}.$$

In other words, the critical nodes of a node v are those nodes interfering with v if the graph G is connected linearly. Based on the results from Section 5.1 we are able to lower-bound the interference of a minimum-interference topology of G as follows.

Lemma 5.5. *Given a graph $G = (V, E)$, let $\gamma = \max_{v \in V} |C_v|$ be the maximum number of critical nodes at any node. A minimum-interference topology for G yields interference in $\Omega(\sqrt{\gamma})$.*

Proof. Let $v \in V$ be the node with maximum interference in G_{lin}. Thus, $|C_v| = \gamma$ as all nodes interfering with v are in C_v. Without loss of generality, we assume that at least half of the nodes in C_v are to the right of v. Let C_v^r be the set of all nodes in C_v to the right of v. We number the nodes $c_i \in C_v^r$ according to their occurrence from left to right. Note that the nodes in C_v^r constitute a *virtual* exponential node chain as the distance to their nearest neighbor to the right must at least double from c_i to c_{i+1}. Therefore, Theorem 5.2 applies directly to the nodes in C_v^r. Due to the fact that $|C_v^r| \geq |C_v|/2$ and together with Theorem 5.2 we obtain $\Omega(\sqrt{|C_v|})$ as a lower bound for the interference at v. □

Algorithm $\mathcal{A}_{apx}$ makes use of Lemma 5.5 in order to decide whether the existing instance exhibits inherently high interference. In particular Algorithm $\mathcal{A}_{apx}$ works as follows: $\mathcal{A}_{apx}$ first computes γ. If $\gamma > \sqrt{\Delta}$, $\mathcal{A}_{gen}$ is applied to the graph. Otherwise, if $\gamma \leq \sqrt{\Delta}$, $\mathcal{A}_{apx}$ connects all nodes of the given graph linearly.

Theorem 5.6. *Given a graph G, Algorithm $\mathcal{A}_{apx}$ computes a resulting topology which approximates the optimal interference of G up to a factor in $\mathrm{O}(\sqrt[4]{\Delta})$.*

Proof. We analyze the two possible cases in $\mathcal{A}_{apx}$.

Case $\gamma > \sqrt{\Delta}$: According to Theorem 5.4, $\mathcal{A}_{gen}$ yields interference in $\mathrm{O}(\sqrt{\Delta})$. On the other hand, by Lemma 5.5, a minimum-interference topology produces at least $\Omega(\sqrt{\gamma})$ interference. We therefore obtain an approximation ratio in $\mathrm{O}(\sqrt{\Delta})/\Omega(\sqrt{\gamma}) \in \mathrm{O}(\sqrt[4]{\Delta})$.

Case $\gamma \leq \sqrt{\Delta}$: By Lemma 5.5, the minimum-interference topology results in interference of at least $\Omega(\sqrt{\gamma})$. Connecting G linearly we obtain interference γ by definition. Consequently, the approximation ratio of $\mathcal{A}_{apx}$ is in $\gamma/\Omega(\sqrt{\gamma}) \in \mathrm{O}(\sqrt[4]{\Delta})$. □

6. Conclusions

In contrast to previous work in the field of topology control, which either aims at implicit interference reduction or is based on a sender-centric interference model that hardly reflects reality, we study in this paper an explicit receiver-centric model of interference. The advantages of this interference model are twofold: On the one hand this definition corresponds to intuition due to its receiver-centricity, particularly modeling interference as an effect occurring at the intended receiver of a message, where collisions actually prevent proper reception. On the other hand this interference model is robust with respect to addition or removal of single nodes, in stark contrast to the previously proposed sender-centric interference model.

Based on our interference model we show that there exist network instances where, to the best of our knowledge, all currently known topology control algorithms (establishing exclusively symmetric connections) fail to effectively confine interference at a low level if required to maintain network connectivity. Led by the observation that already one-dimensional networks exhibit the main complexity of finding low-interference connectivity-preserving topologies, we then focus on the so-called highway model. Starting out to study the special case of the exponential node chain, we finally obtain an algorithm that is guaranteed to always compute a $\sqrt[4]{\Delta}$-approximation of the optimal connectivity-preserving topology in the highway model in general.

Adaptation of our approach to higher dimensions remains an open problem and is left for future work.

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